Hierarchical community structure in networks

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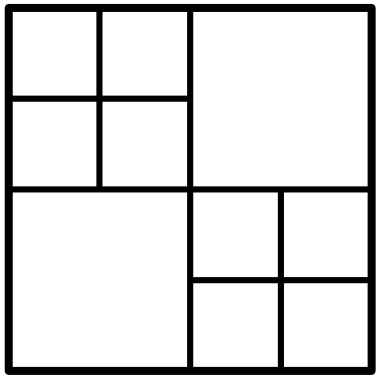
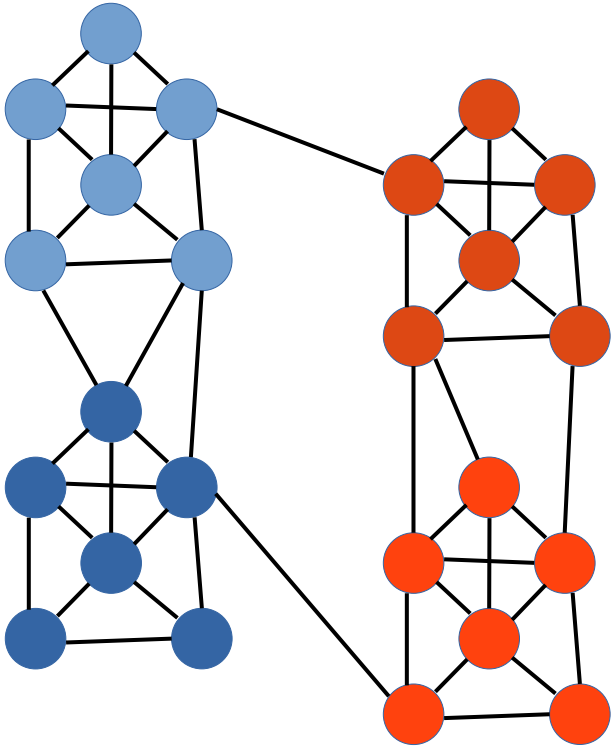
Pre-print available

arXiv:2009.07196

In collaboration with Michael Schaub (Aachen)
and Jiaze Li (Maastricht)

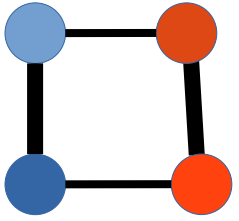
Building the hierarchy





Observed network

infer
communities

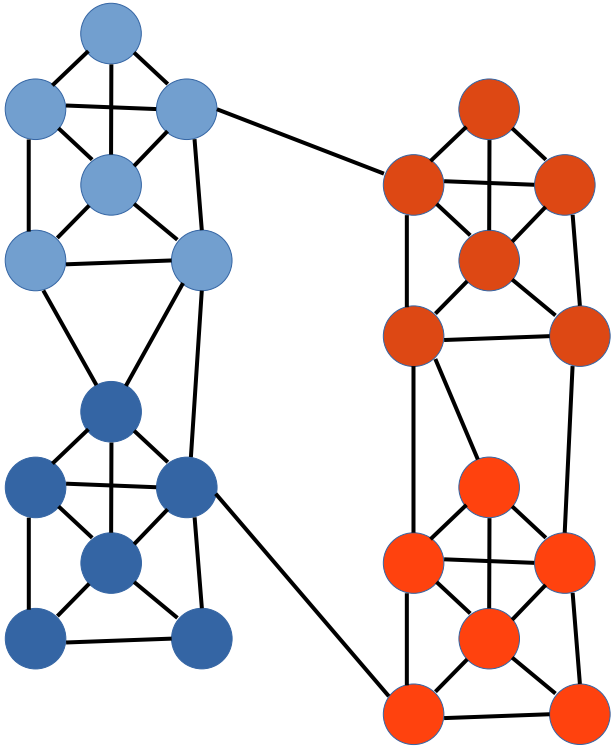


Multigraph

infer
communities

Multigraph

Building the hierarchy



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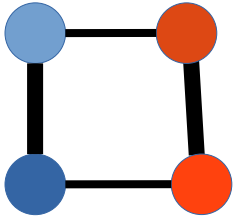
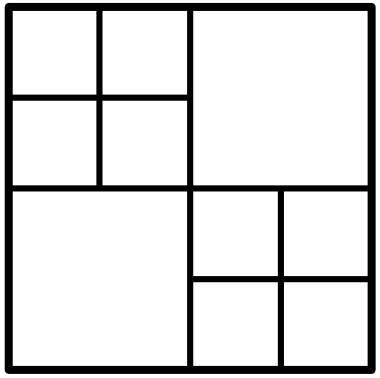


Multigraph

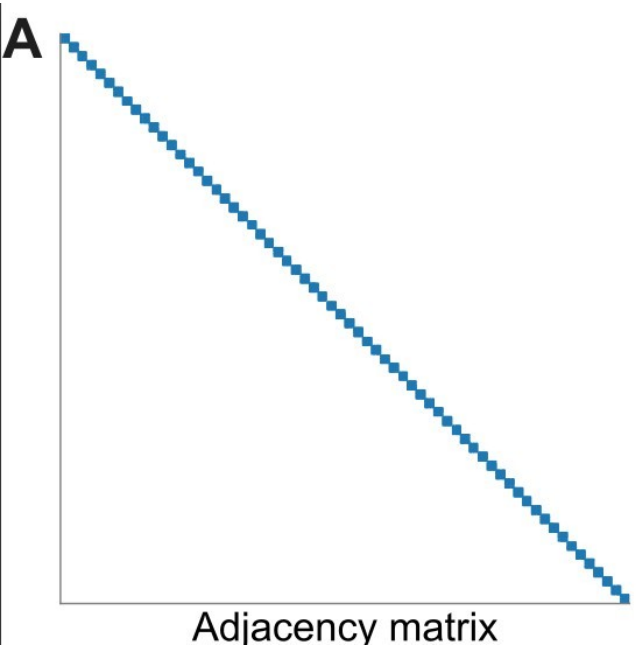
infer communities

Multigraph

Is this hierarchy any good?



Is there a hierarchy?



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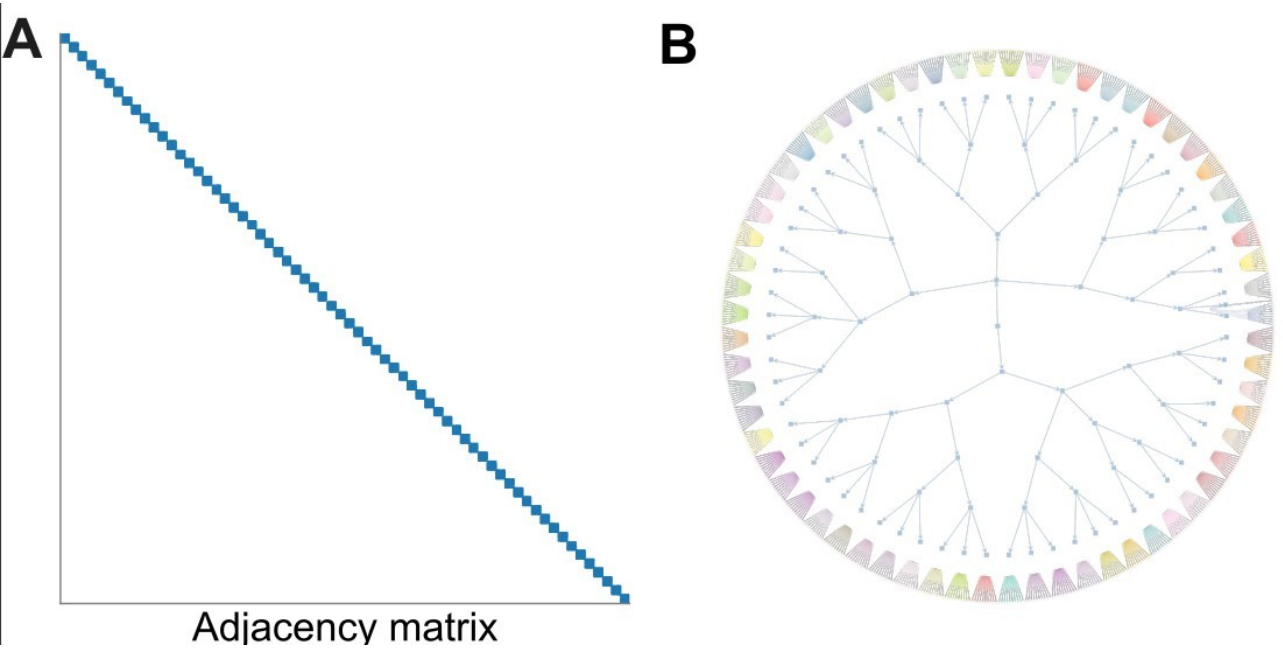

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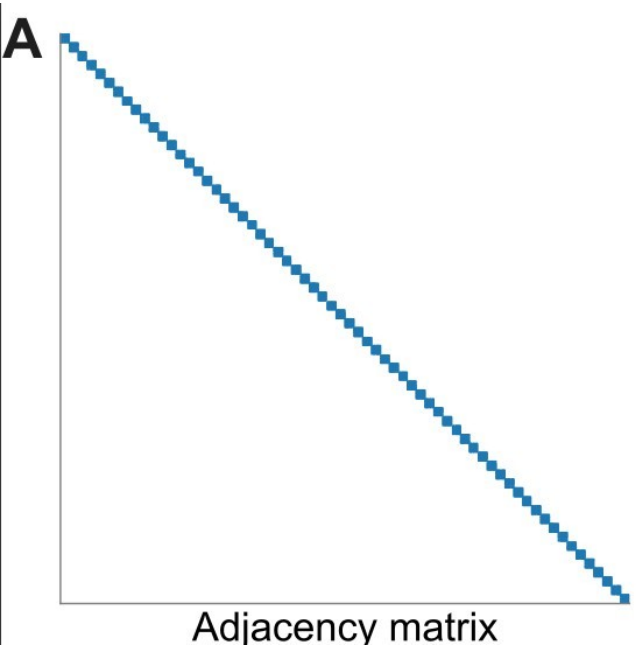
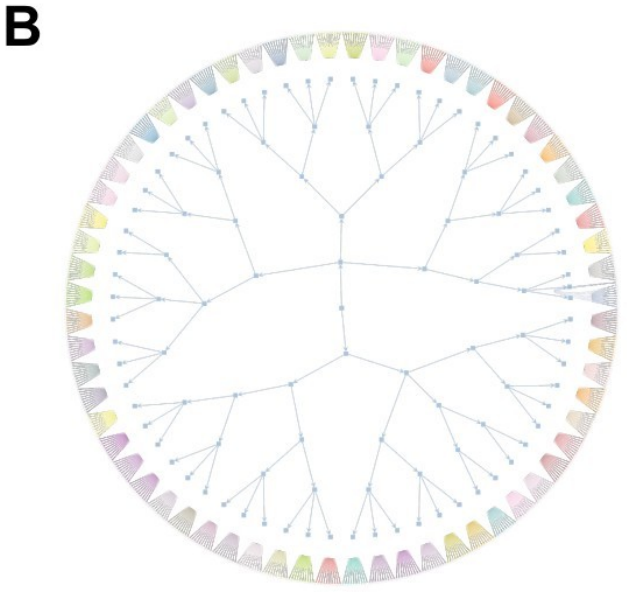
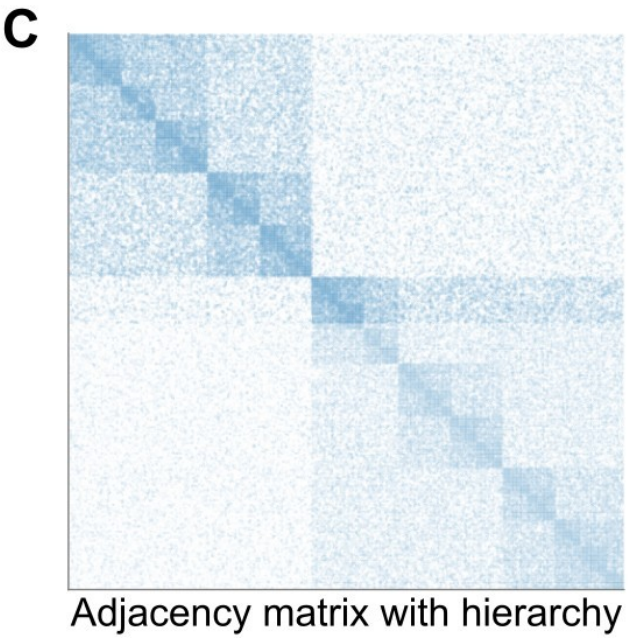
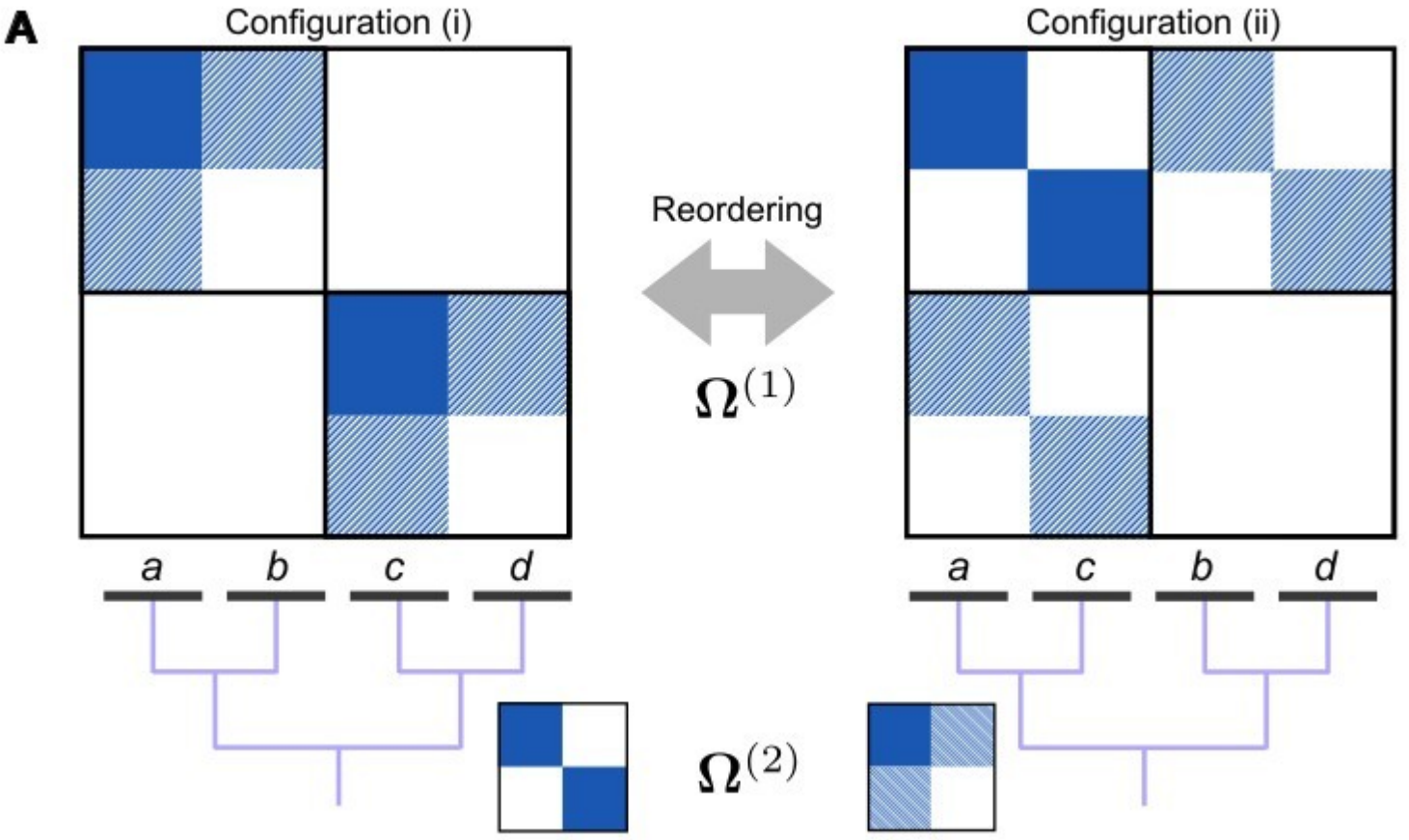






Is the hierarchy identifiable?



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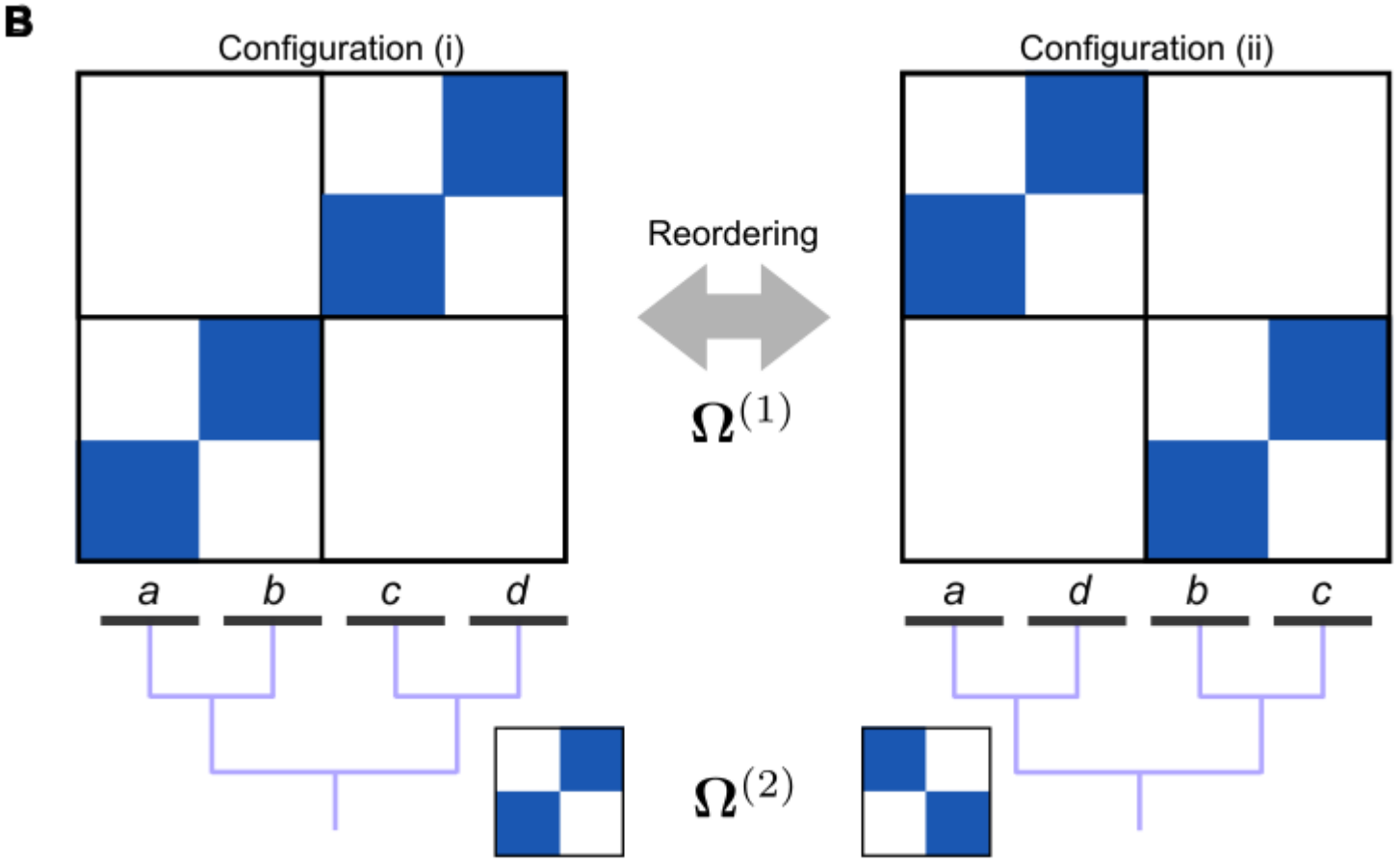
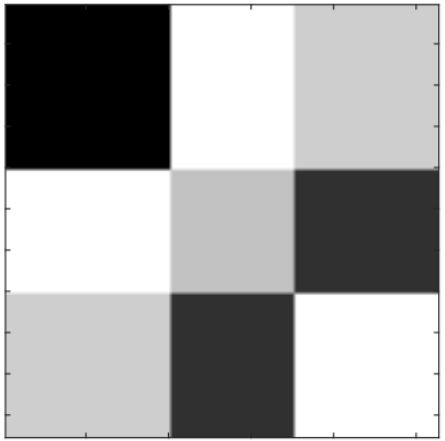
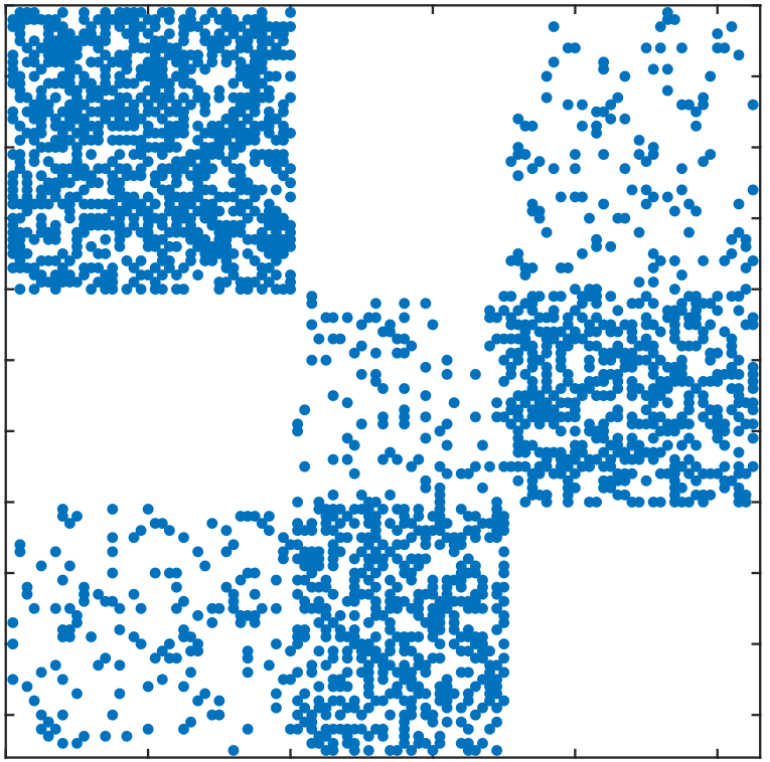


The Stochastic Blockmodel (SBM)



generation

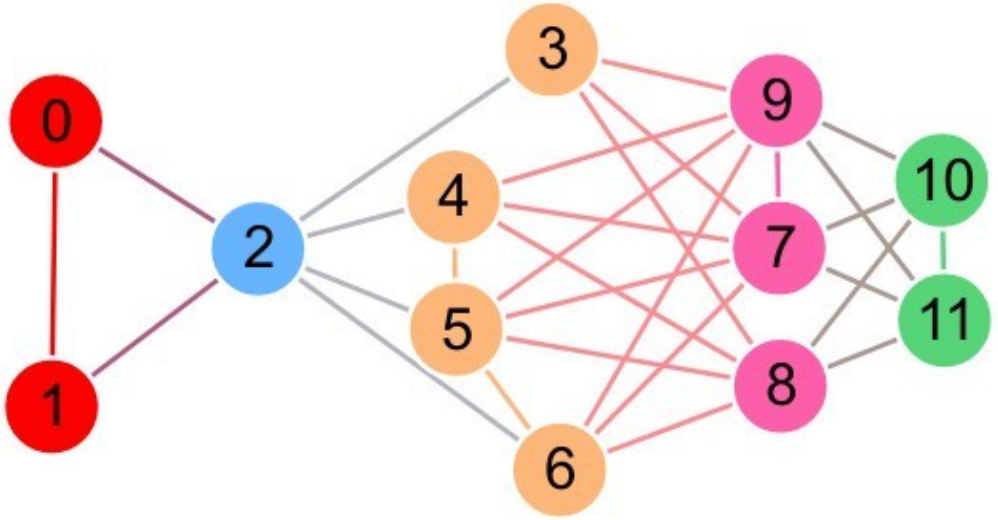
inference

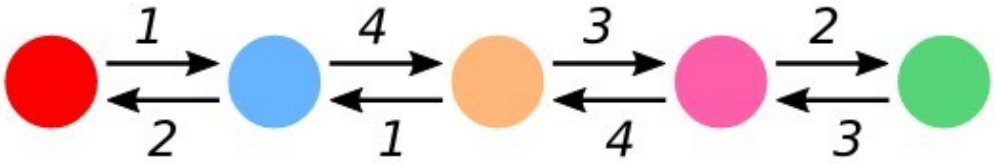


Mixing Matrix

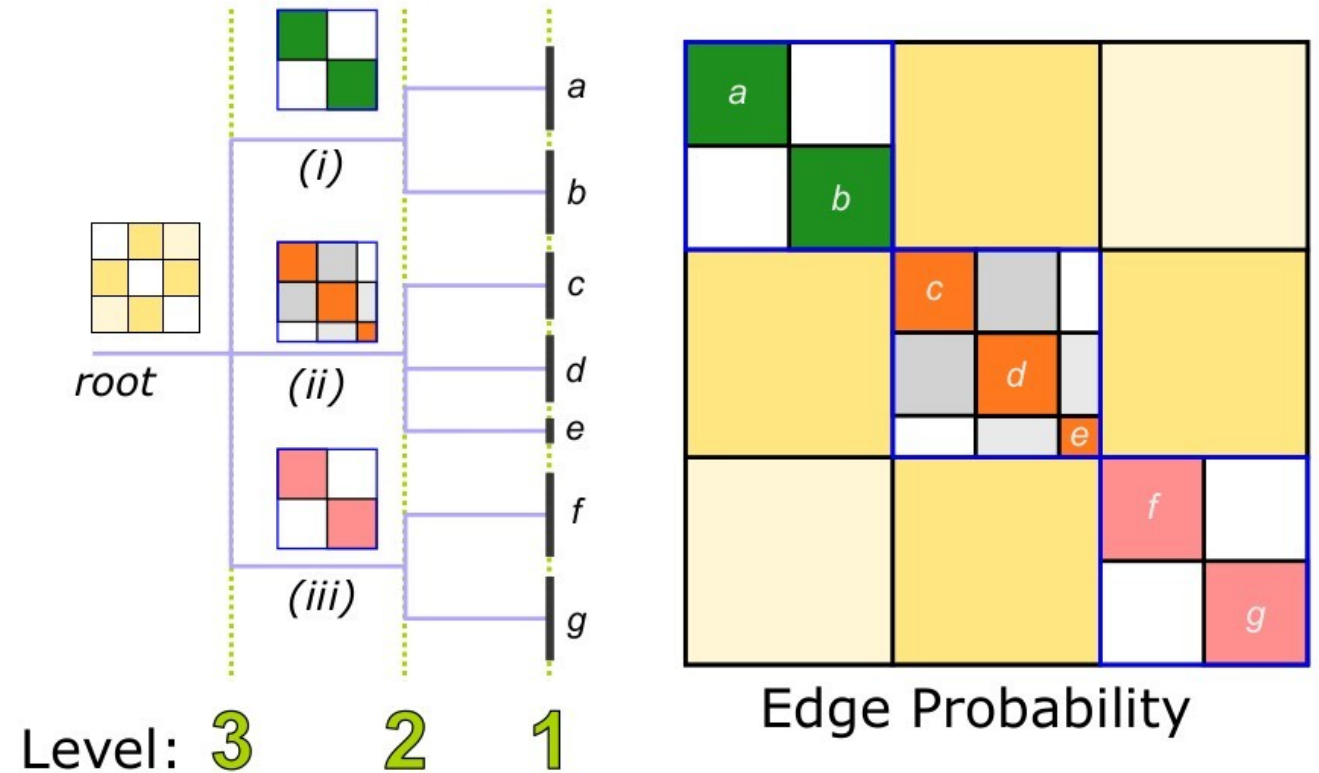
Adjacency Matrix

External equitable partitions (EEPs)





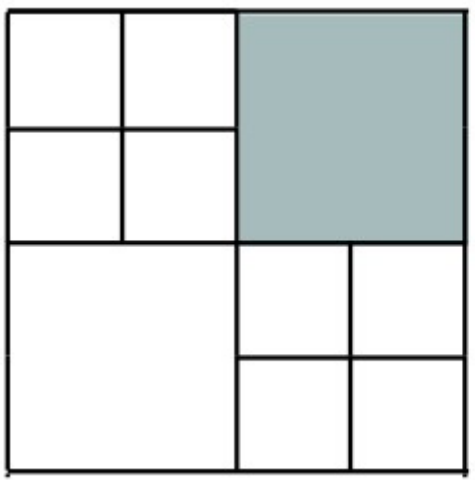
Expected adjacency has an EEP at each hierarchical level



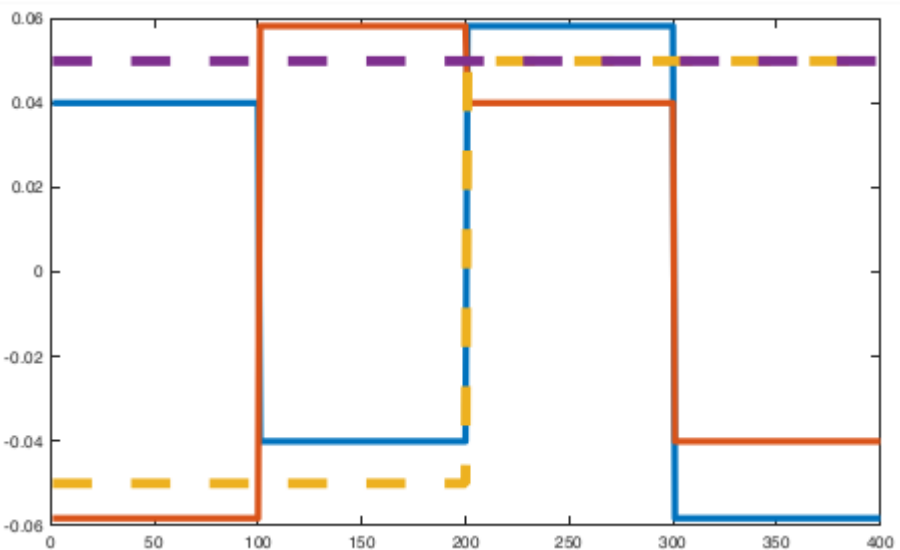
Constant probability between groups at each hierarchical level

Spectral properties

$\mathbb{E}[A]$

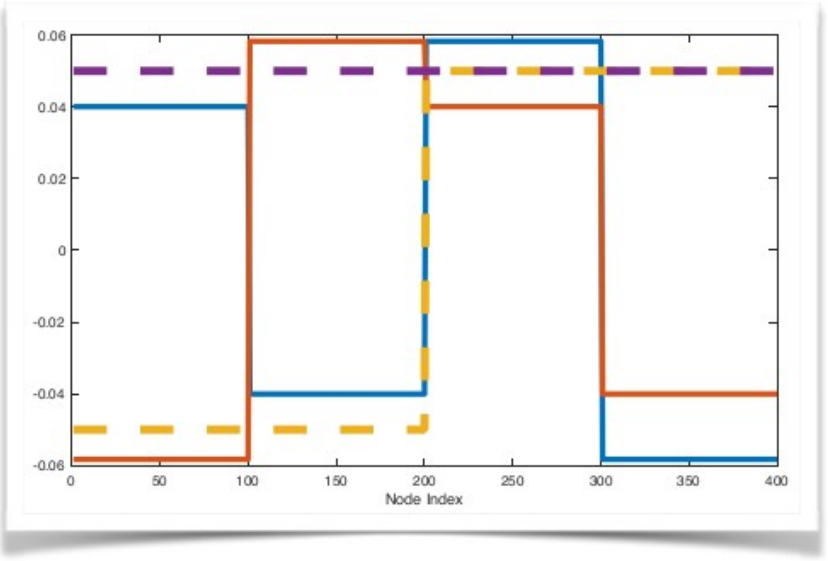


First 4 Eigenvectors of the Laplacian



Node index

If we could just “see” the expected adjacency matrix, then we could just look for constant eigenvectors



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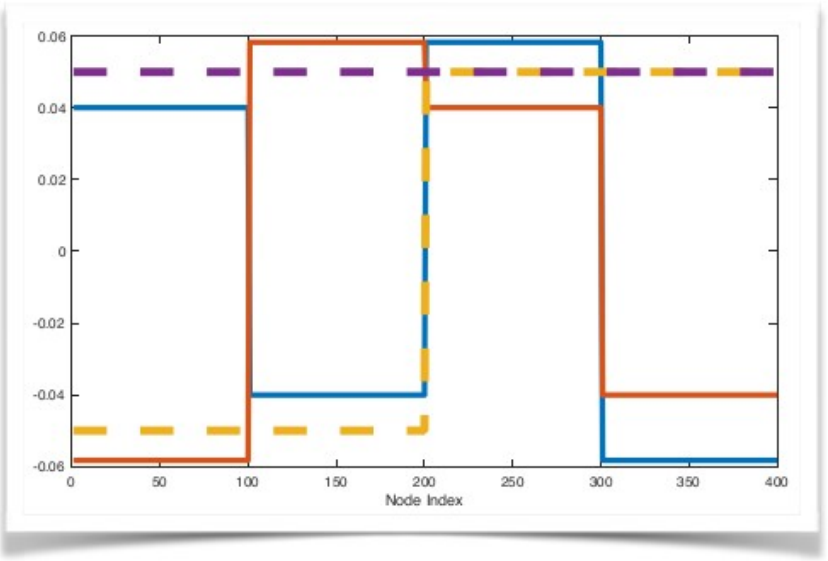
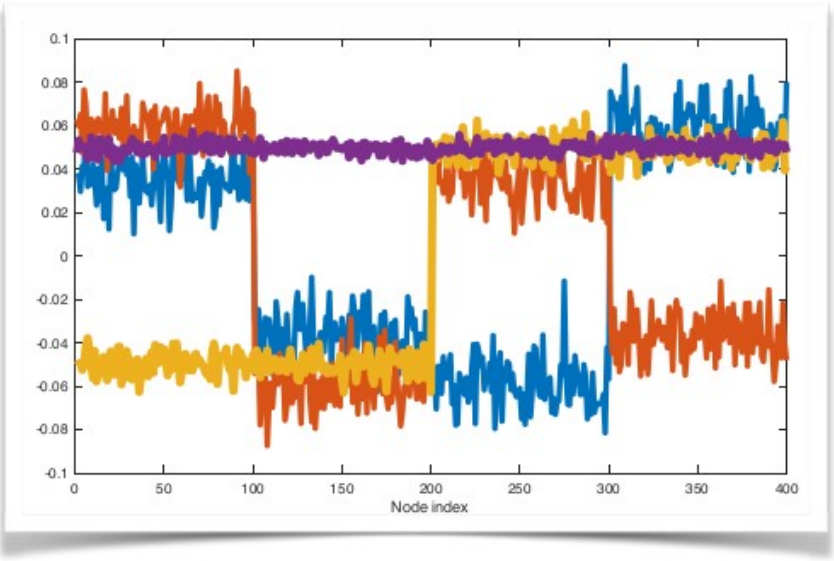
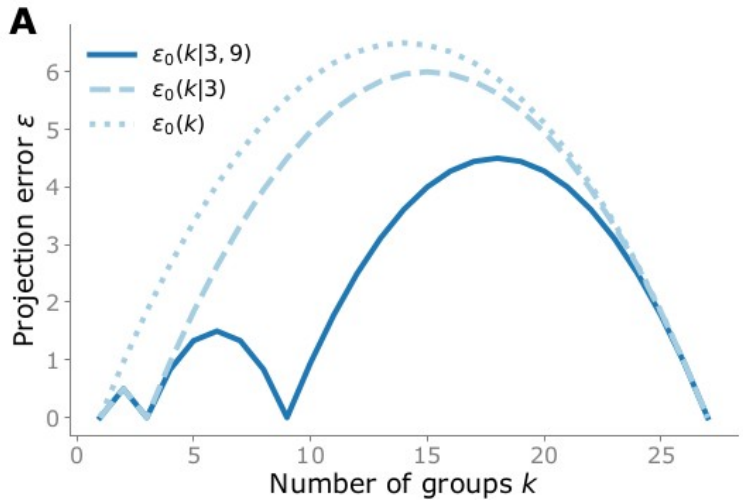
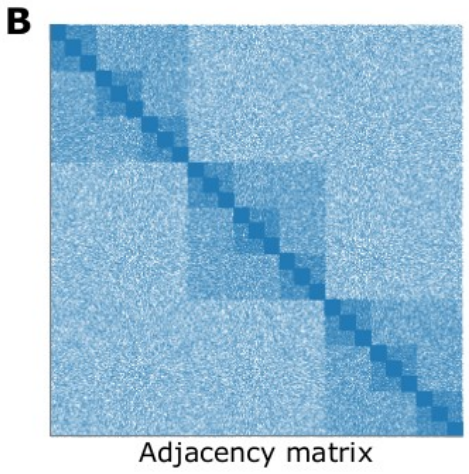


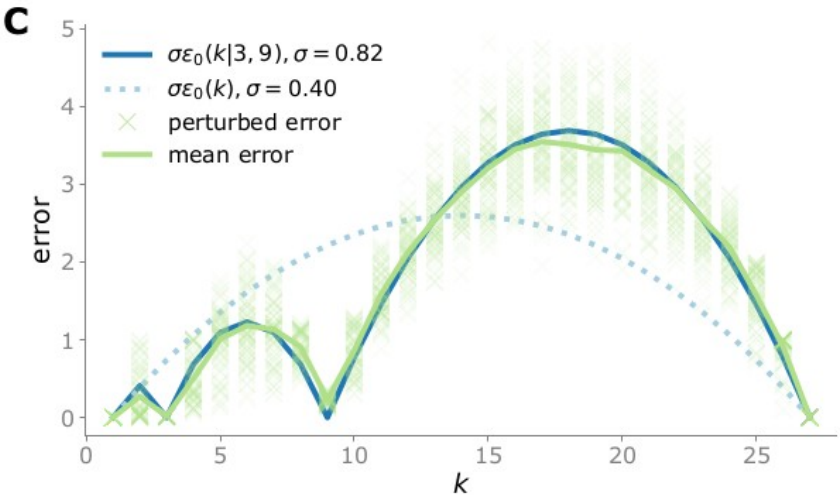


Eigenvector variance
within groups yields a
projection error

Instead, we compare the expected projection error with the mean error of randomly perturbations of the network.



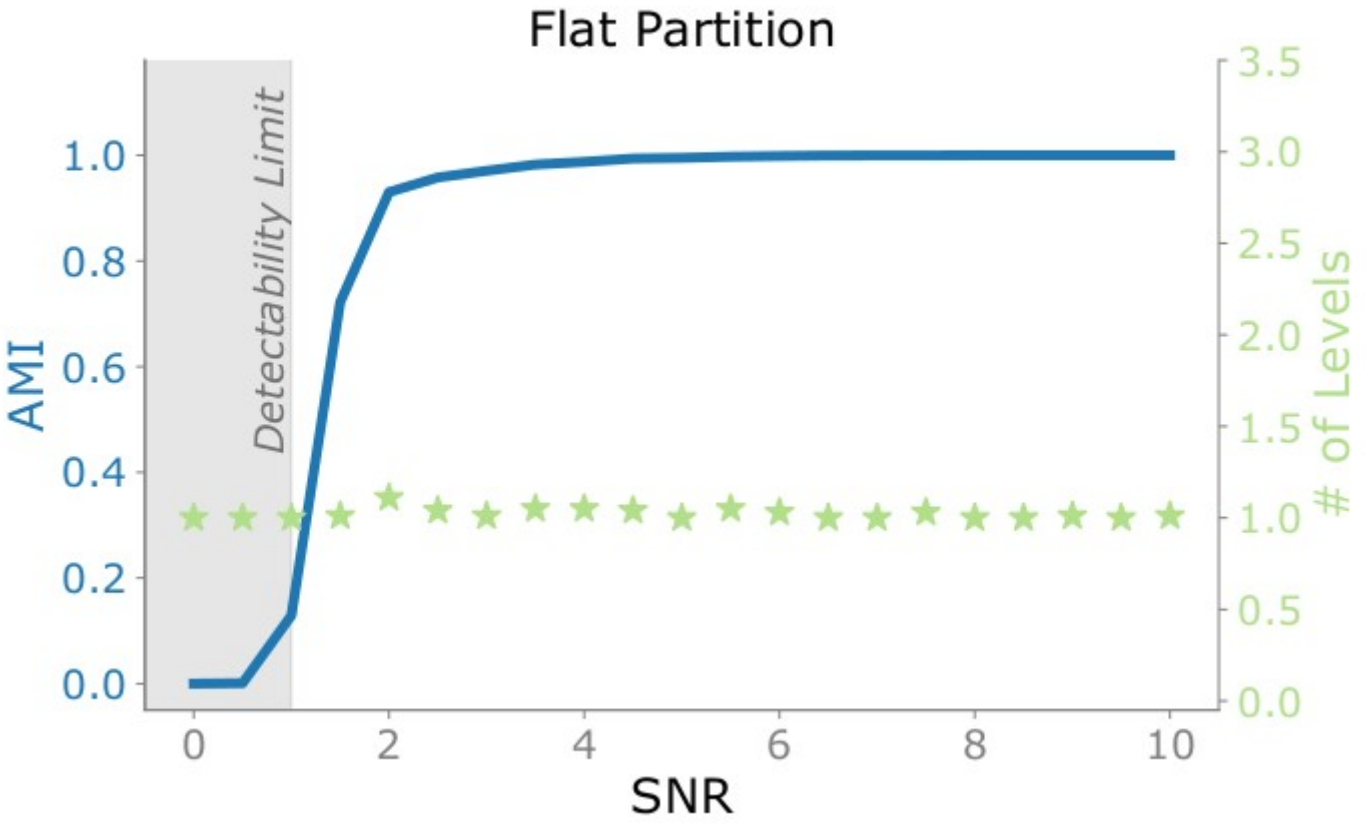




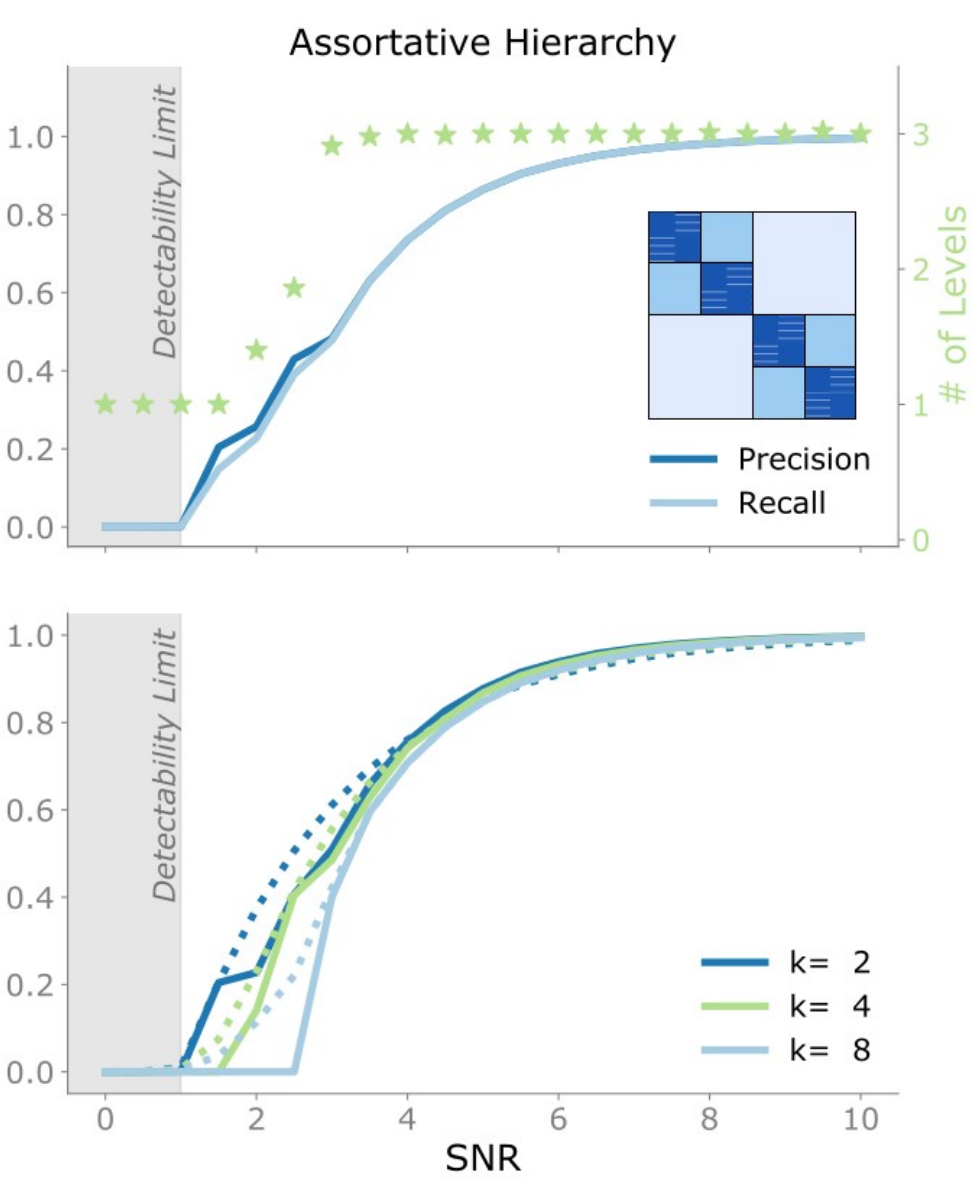
Expected error

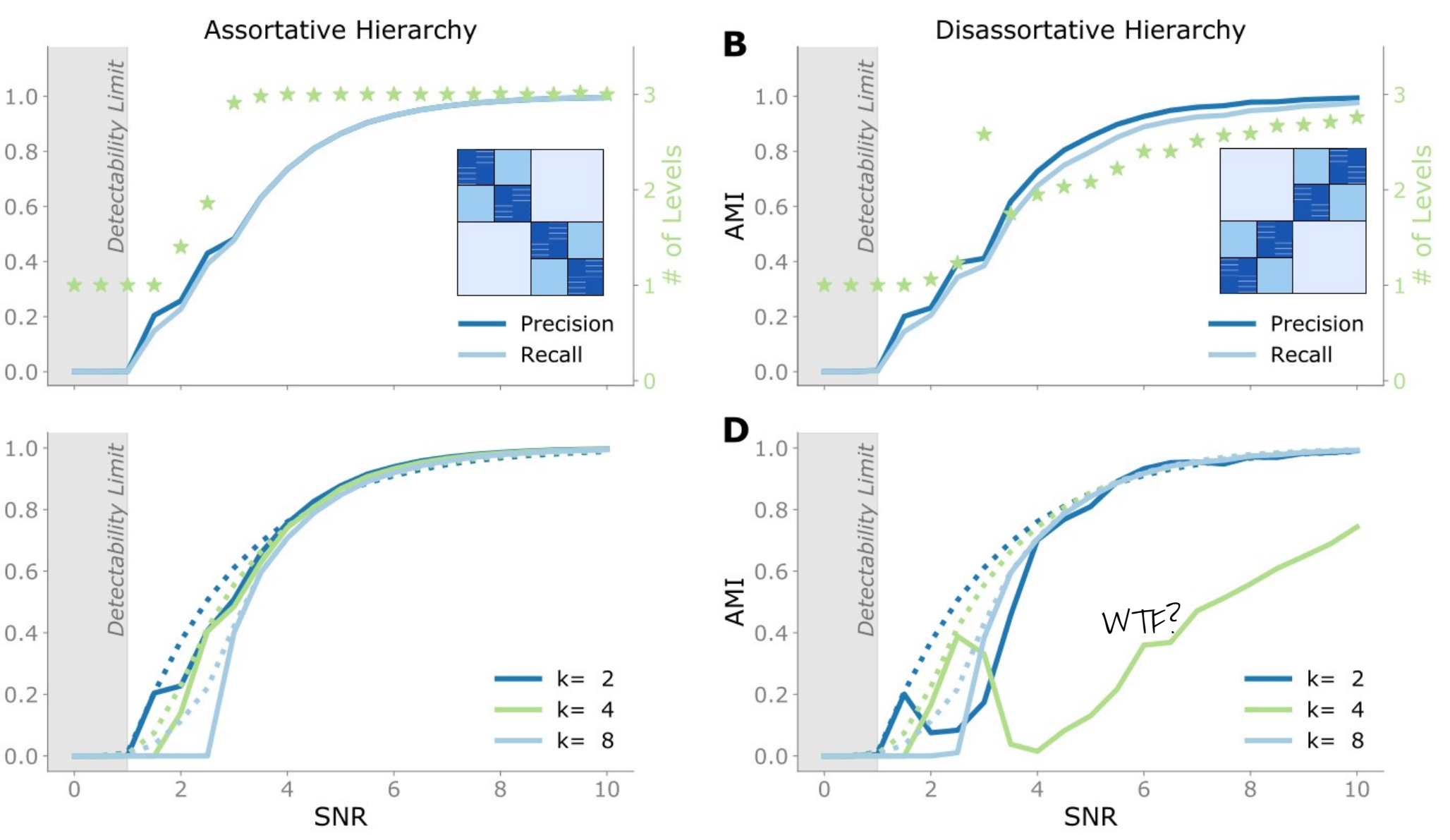
Mean perturbed error

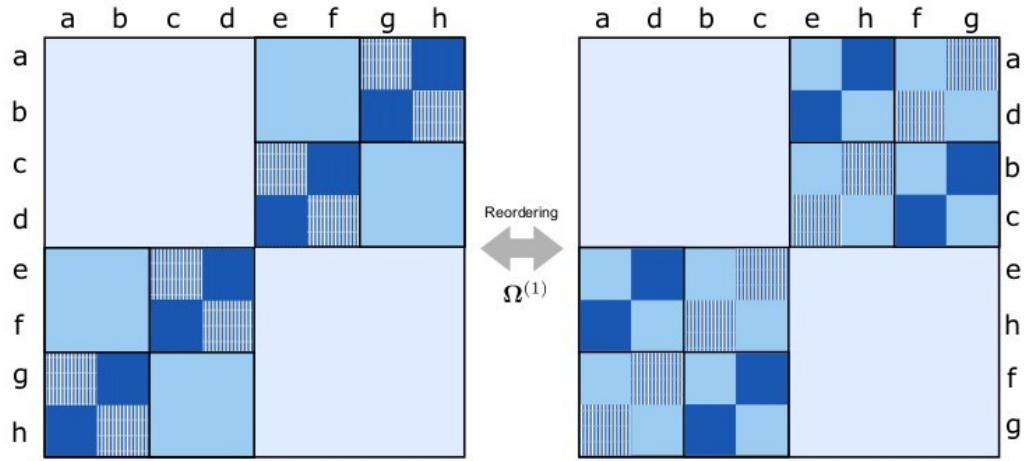
It's fast, but does it work?

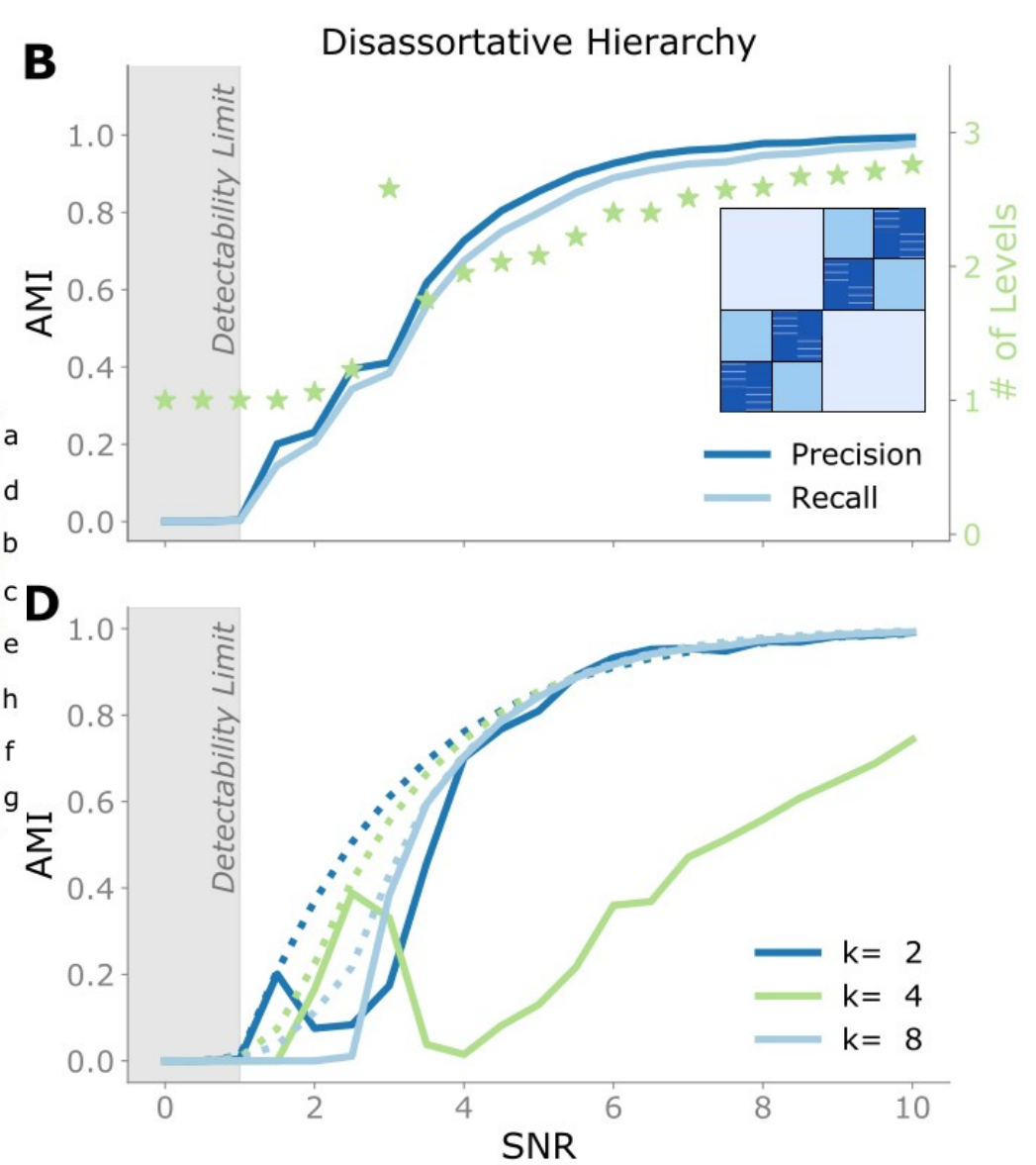


(Signal-to-noise ratio)


Assortative Hierarchy
Detectability Limit
1.0
0.8
0.6
0.4
0.2
0.0
3
2
1
0
# of Levels
Precision
Recall
Detectability Limit
k= 2
k= 4
k= 8
0
2
4
6
8
10
SNR


Assortative Hierarchy
Disassortative Hierarchy
B
D
Detectability Limit
AMI
# of Levels
Precision
Recall
k= 2
k= 4
k= 8
SNR
WTF?


a b c d e f g h
a b c d e f g h
Reordering
$\Omega^{(1)}$
a d b c e h f g
a d b c e h f g


B
Disassortative Hierarchy
Detectability Limit
AMI
# of Levels
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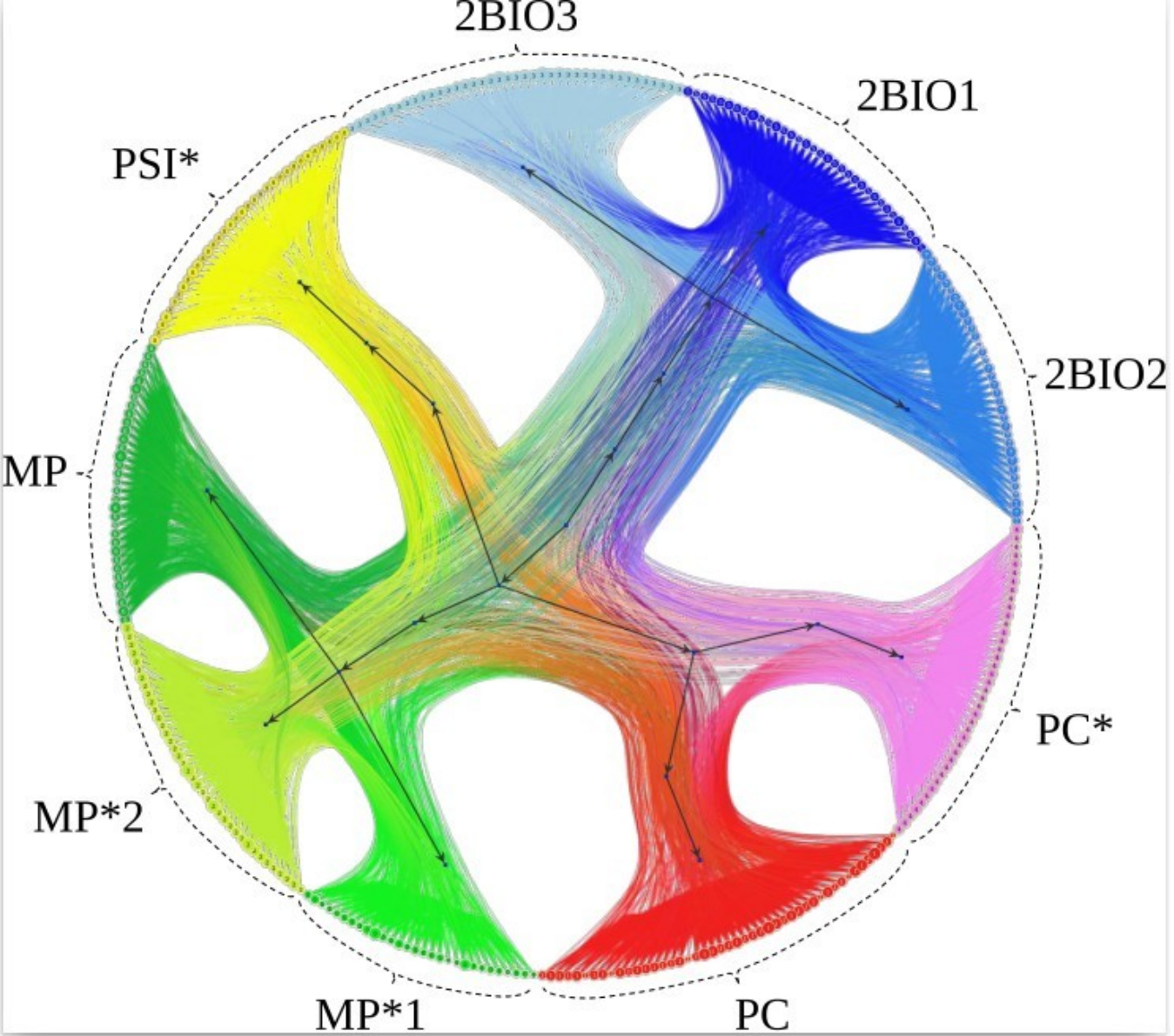
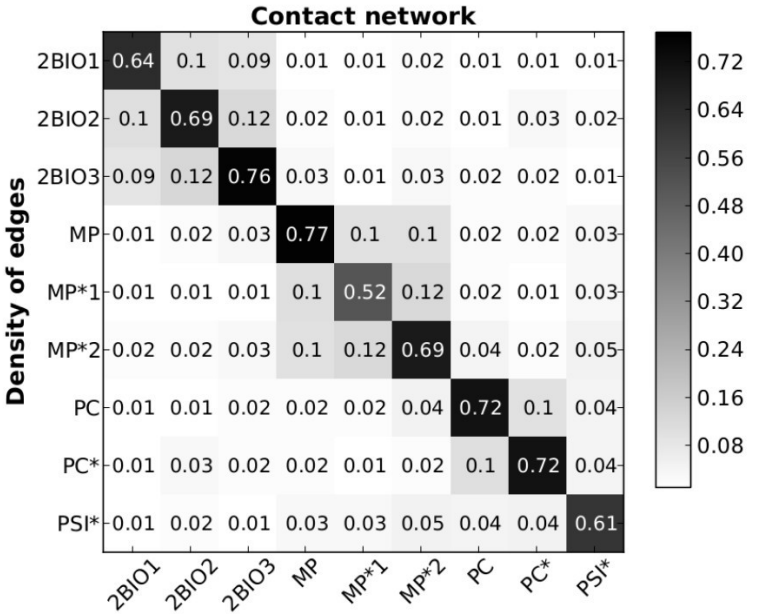
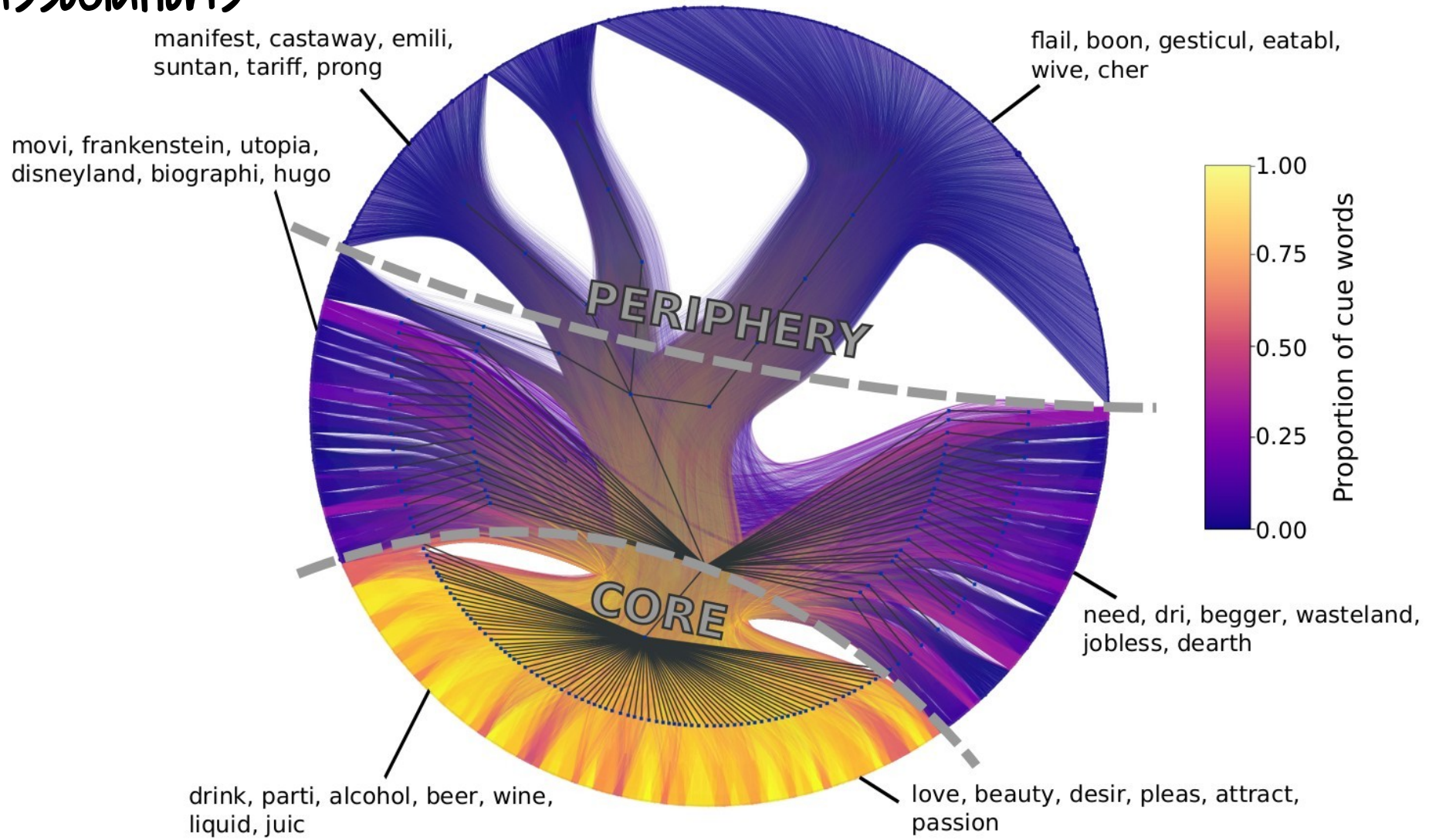
Recall
D
Detectability Limit
AMI
SNR
k= 2
k= 4
k= 8



Face-to-face contacts



Word associations



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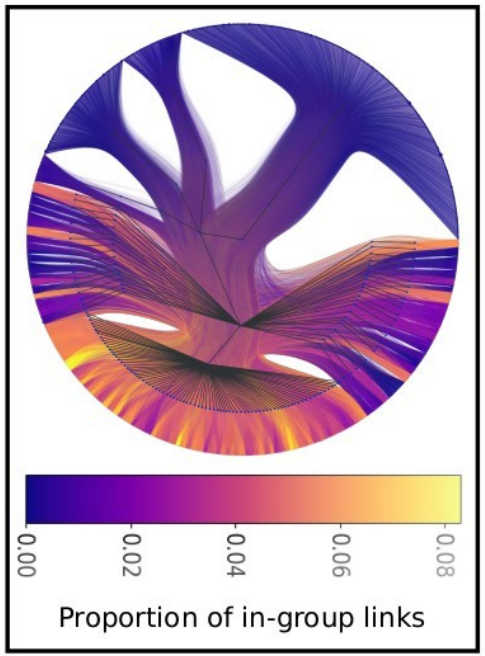

manifest, castaway, emili, suntan, tariff, prong

flail, boon, gesticul, eatabl, wive, cher

movi, frankenstein, utopia, disneyland, biographi, hugo

PERIPHERY

CORE

1.00
0.75
0.50
0.25
0.00
Proportion of cue words

need, dri, begger, wasteland, jobless, dearth

drink, parti, alcohol, beer, wine, liquid, juic

love, beauty, desir, pleas, attract, passion

Take home messages...

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- Hierarchies provide a multi-resolution summary
  - descriptions at many different levels

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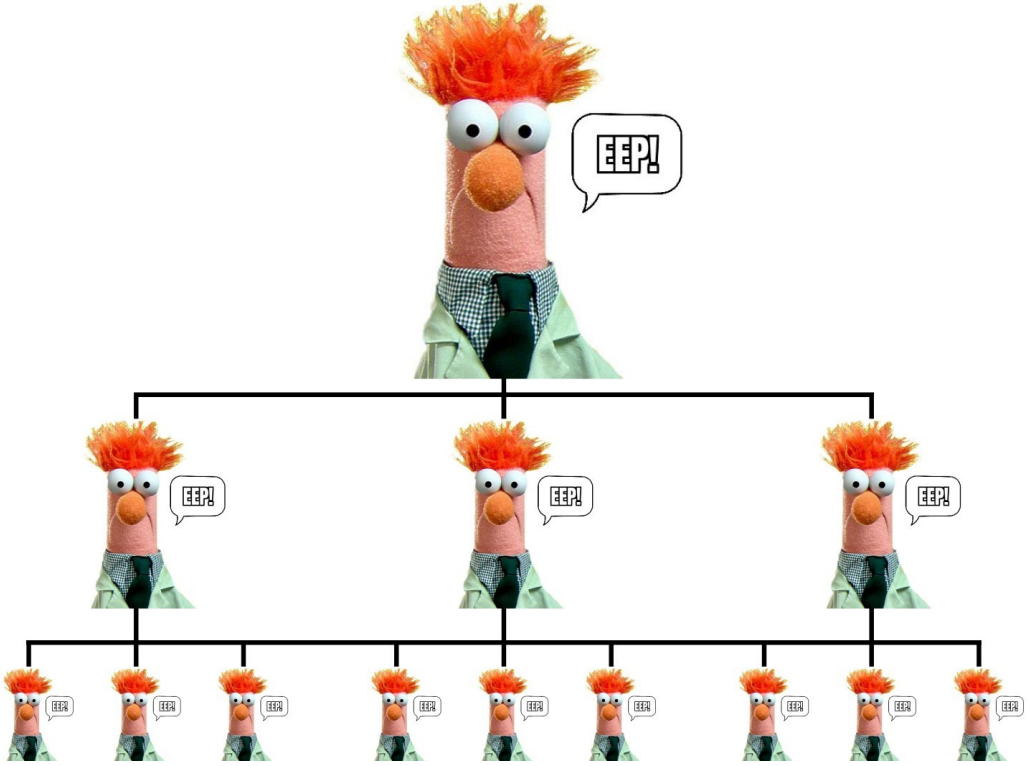
- sEEP as a framework to conceptualise hierarchies
- Spectral methods for efficient hierarchical community detection

In collaboration with

Michael Schaub
(RWTH Aachen)

Jiaze Li
(UM)



"It's EEPs all the way down"

Pre-print available

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Contact:
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@PiratePeel