

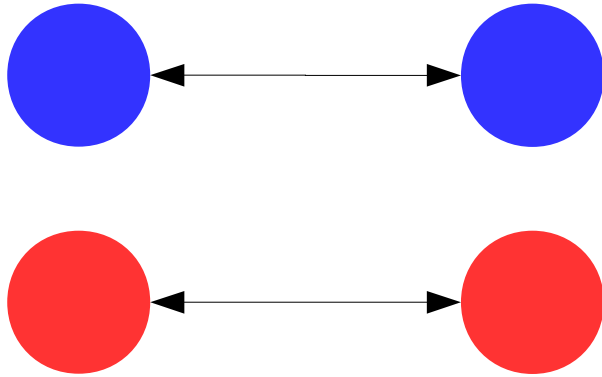
# Multi-scale mixing in complex networks

Leto Peel

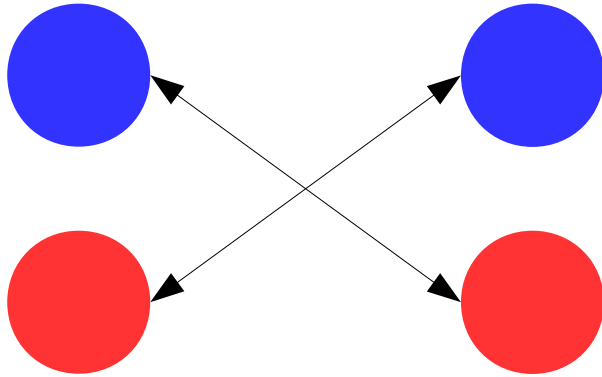
Université catholique de Louvain

@PiratePeel

# Assortativity



# Disassortativity



# Mixing in real networks



# Mixing in real networks



# Mixing in real networks

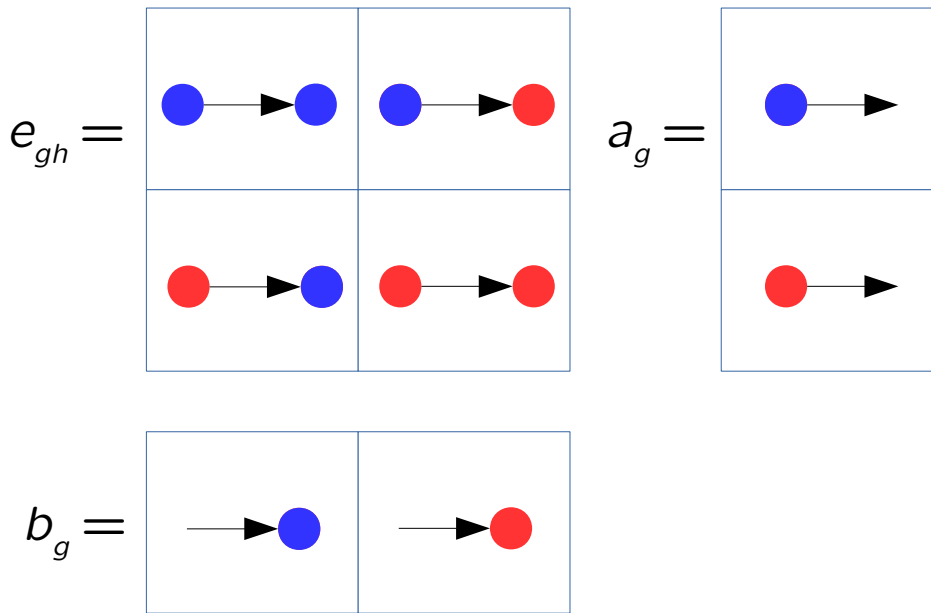




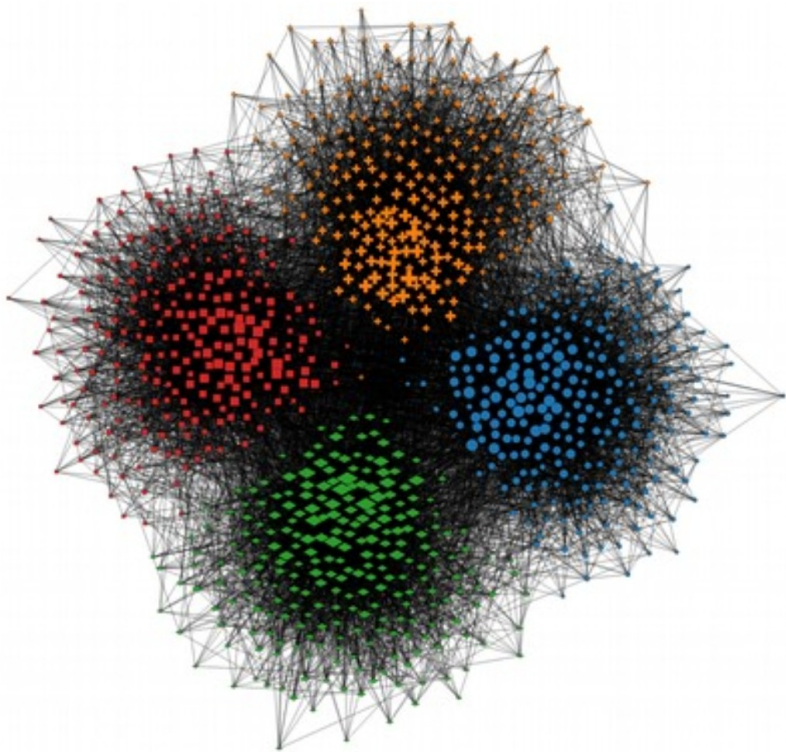
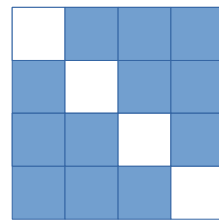
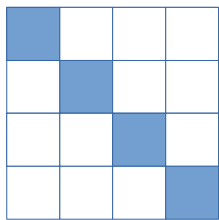
# Generalisation, not rules!



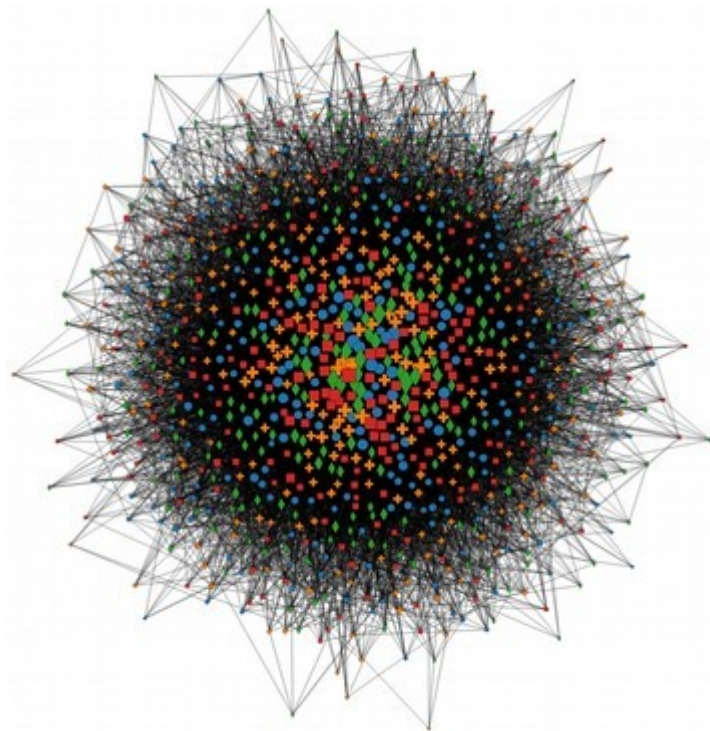
$$r_{\text{global}} = \frac{\sum_g e_{gg} - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$







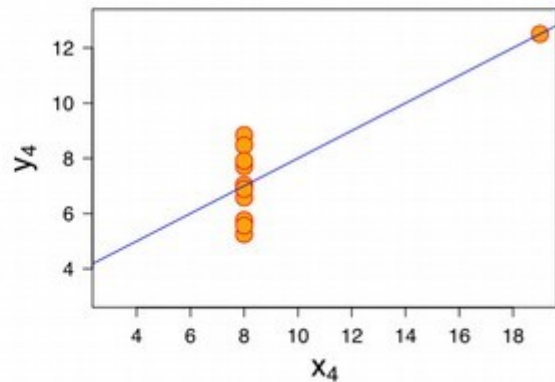
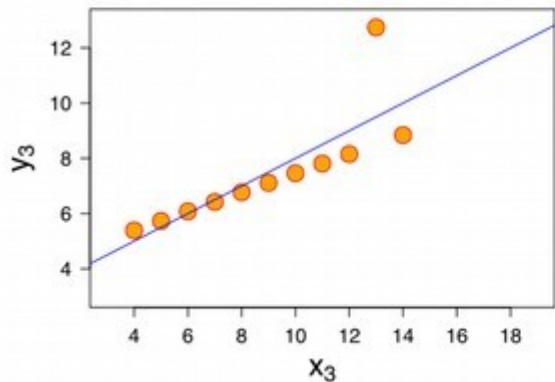
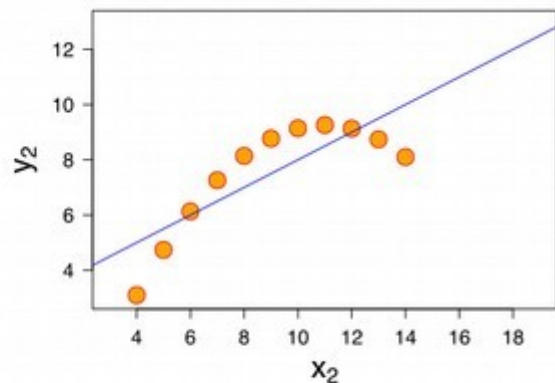
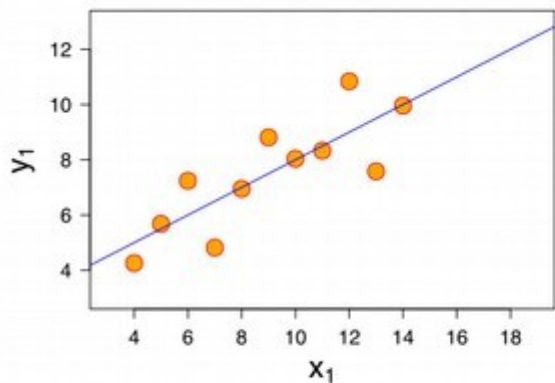
$r \sim 0.7$



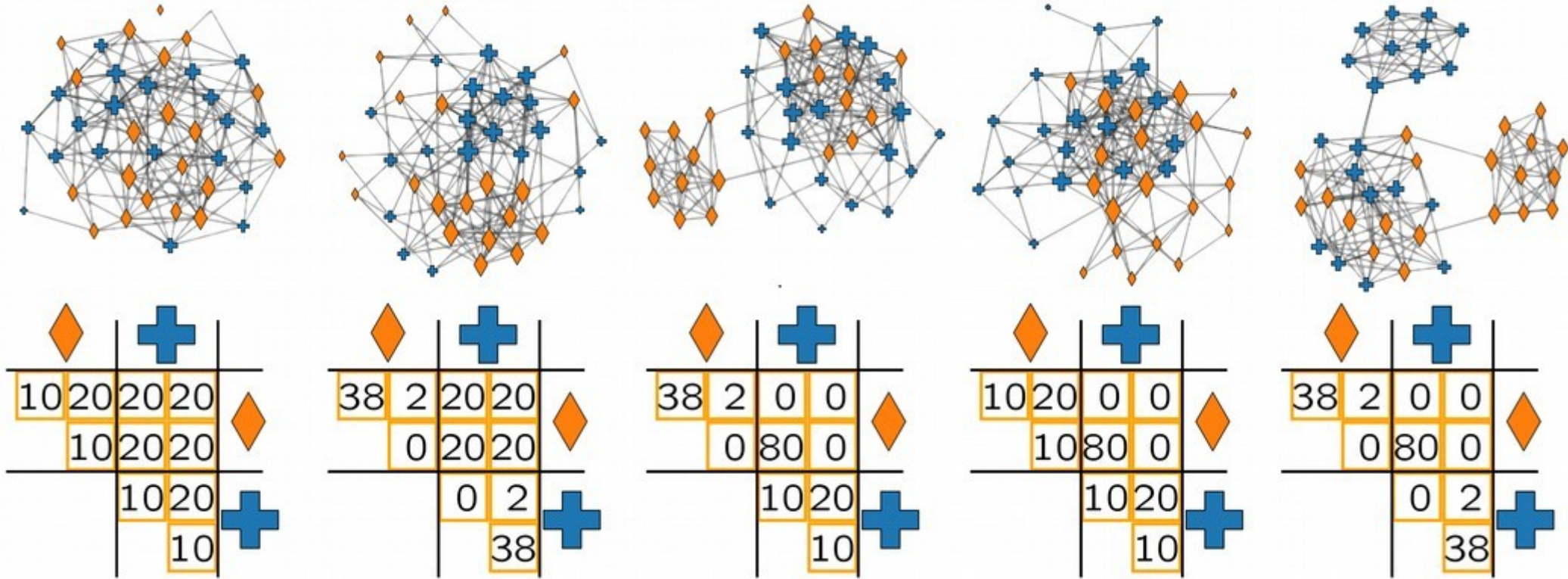
$r \sim -0.3$

Assortativity is correlation across edges

# Assortativity is correlation across edges



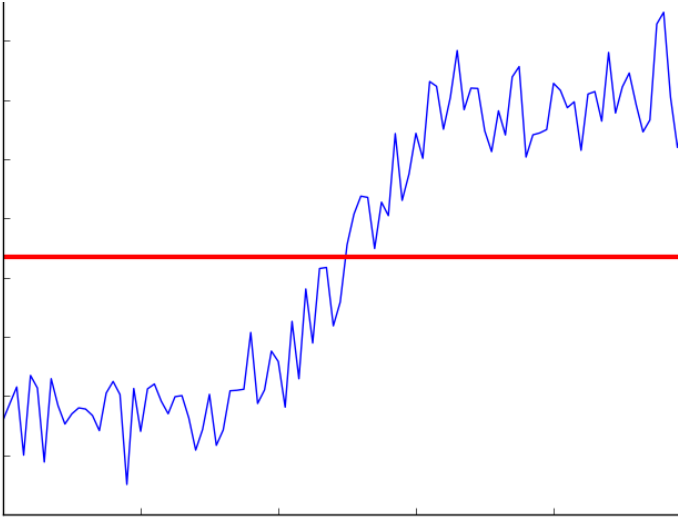
All these networks have assortativity  $r=0$



Can we measure assortativity locally?

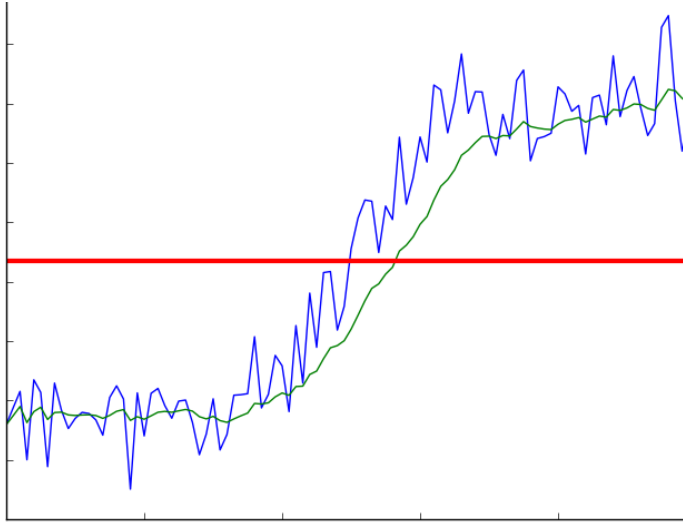


# Time series analysis



The mean is only representative of the data around the middle of the time series

# Time series analysis



Exponentially weighted mean

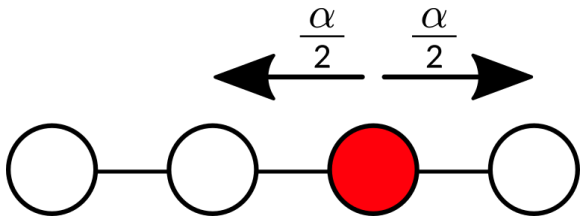
Recent points are more relevant

$$S_t = \alpha y_{t-1} + (1 - \alpha)S_{t-1} \quad 0 < \alpha \leq 1$$

# Random walks on a graph



$$e_{gh} = \sum_{ij} w(i) \frac{A_{ij}}{k_i} \delta_{y_i, g} \delta_{y_j, h}$$



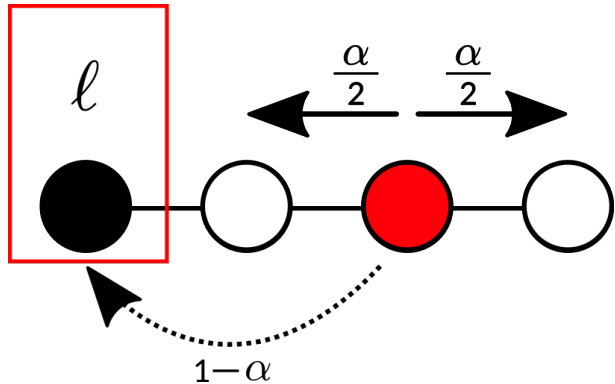
$$w(i; \ell)$$

stationary distribution  
of a random walk  
(PageRank)

# Random walks on a graph



$$e_{gh}(\alpha, \ell) = \sum_{ij} w(i; \ell) \frac{A_{ij}}{k_i} \delta_{y_i, g} \delta_{y_j, h}$$



$$w(i) = \frac{k_i}{2m}$$

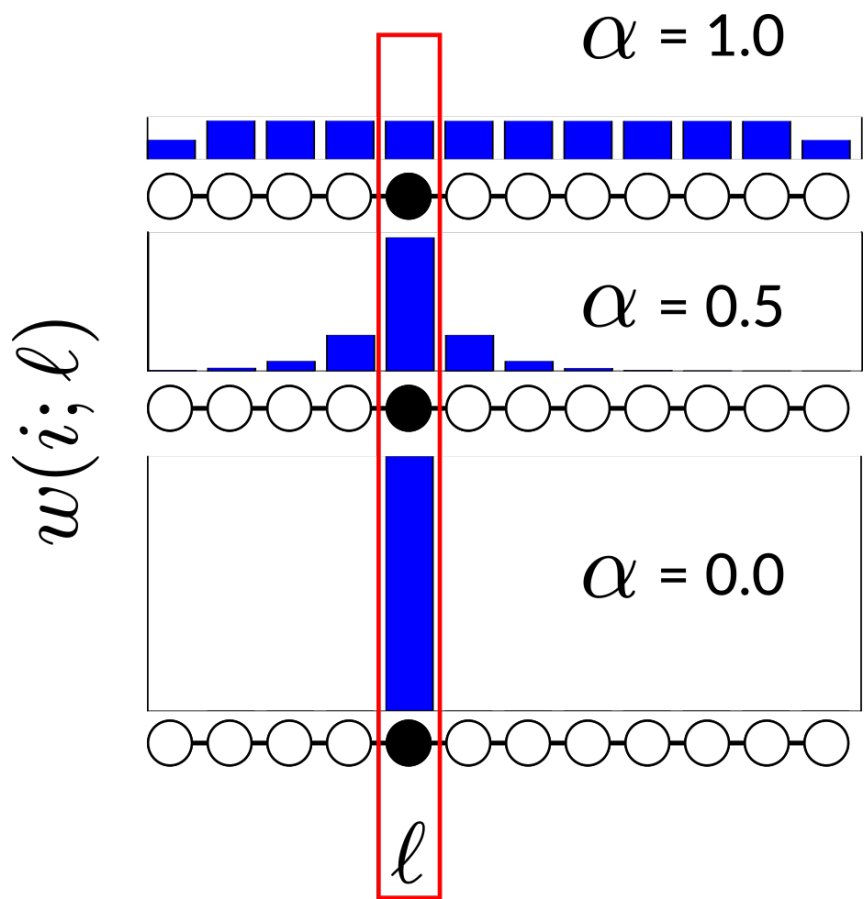
stationary distribution  
of a random walk with  
restart  
(Personalised PageRank)

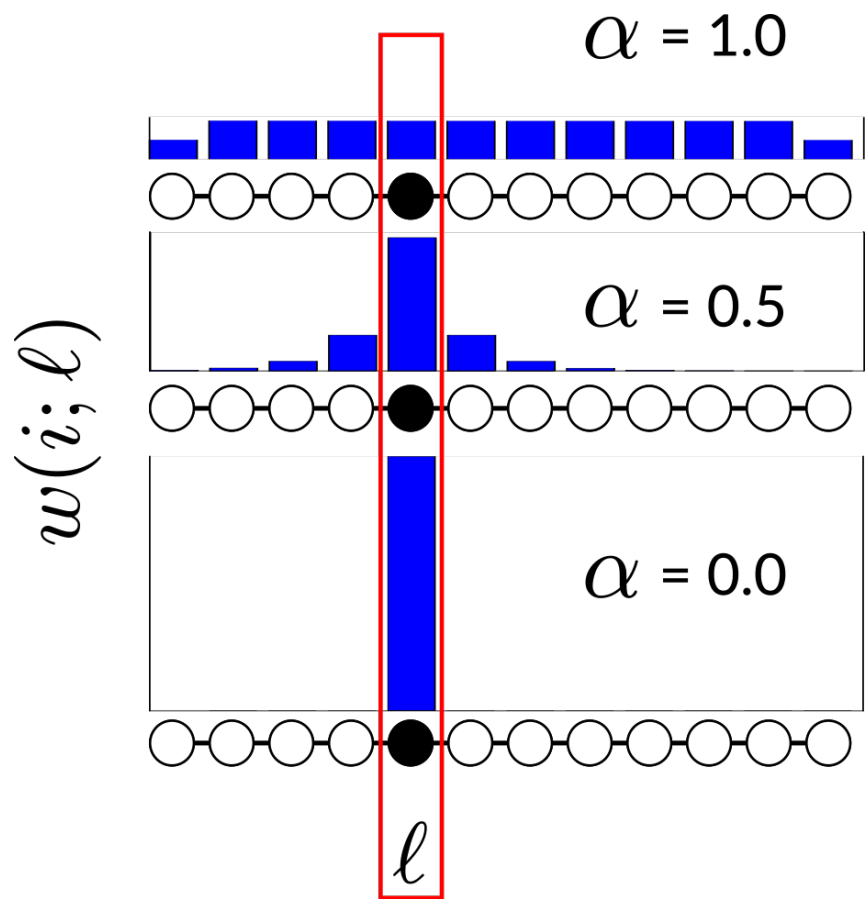
# Random walks on a graph

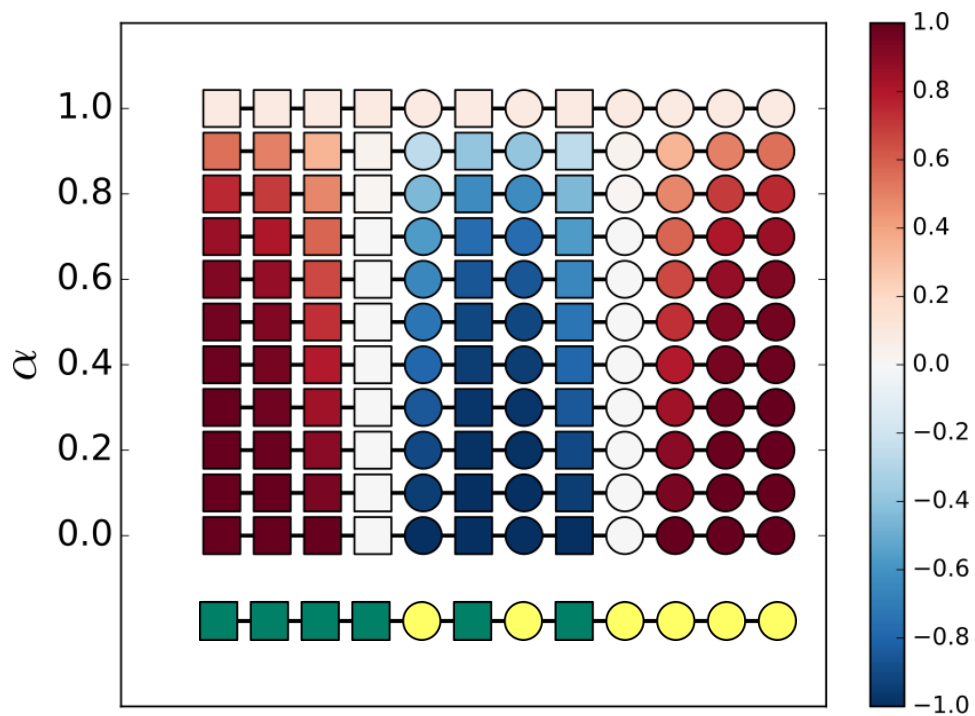
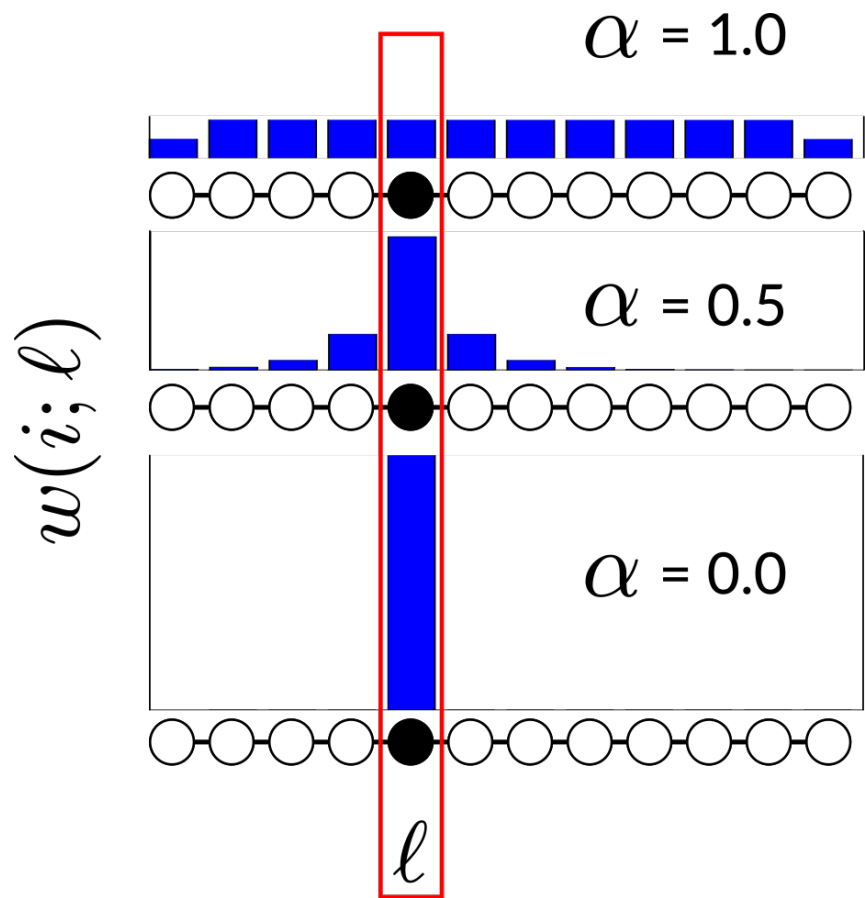
$$e_{gh}(\alpha, \ell) = \sum_{ij} w(i; \ell) \frac{A_{ij}}{k_i} \delta_{y_i, g} \delta_{y_j, h}$$

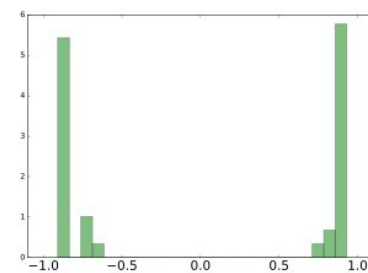
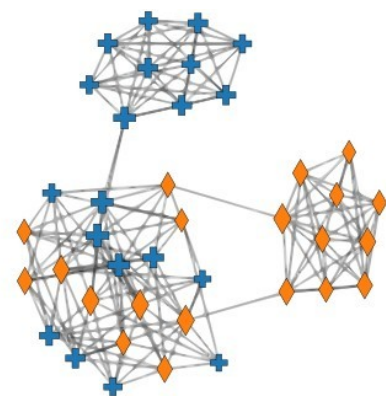
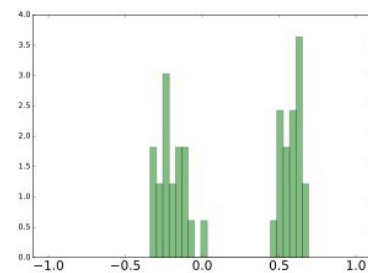
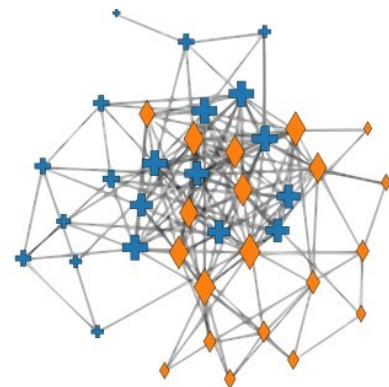
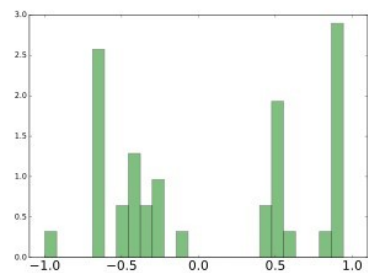
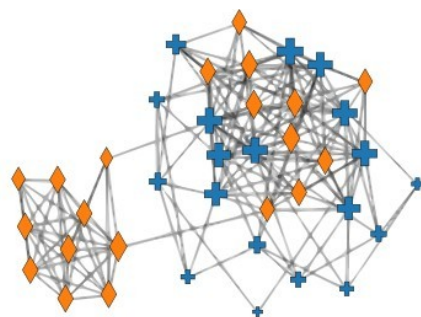
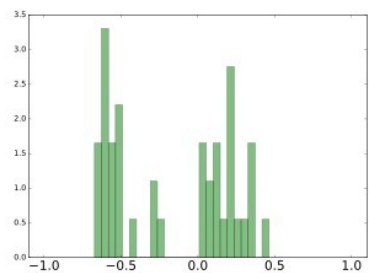
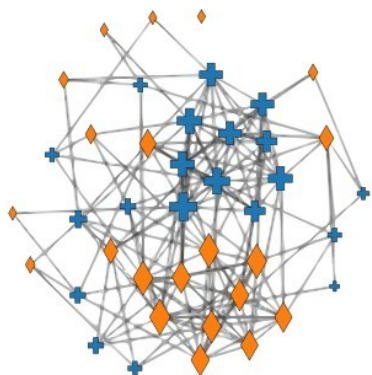
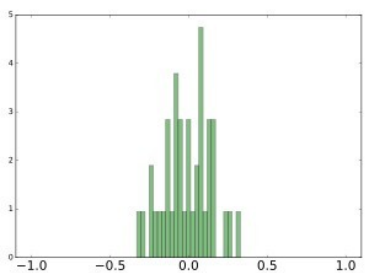
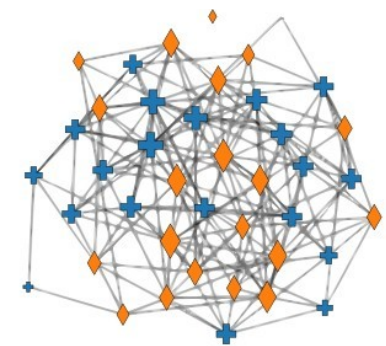
$$r(\alpha, \ell) = \frac{\sum_g e_{gg}(\alpha, \ell) - \sum_g a_g b_g}{1 - \sum_g a_g b_g}$$



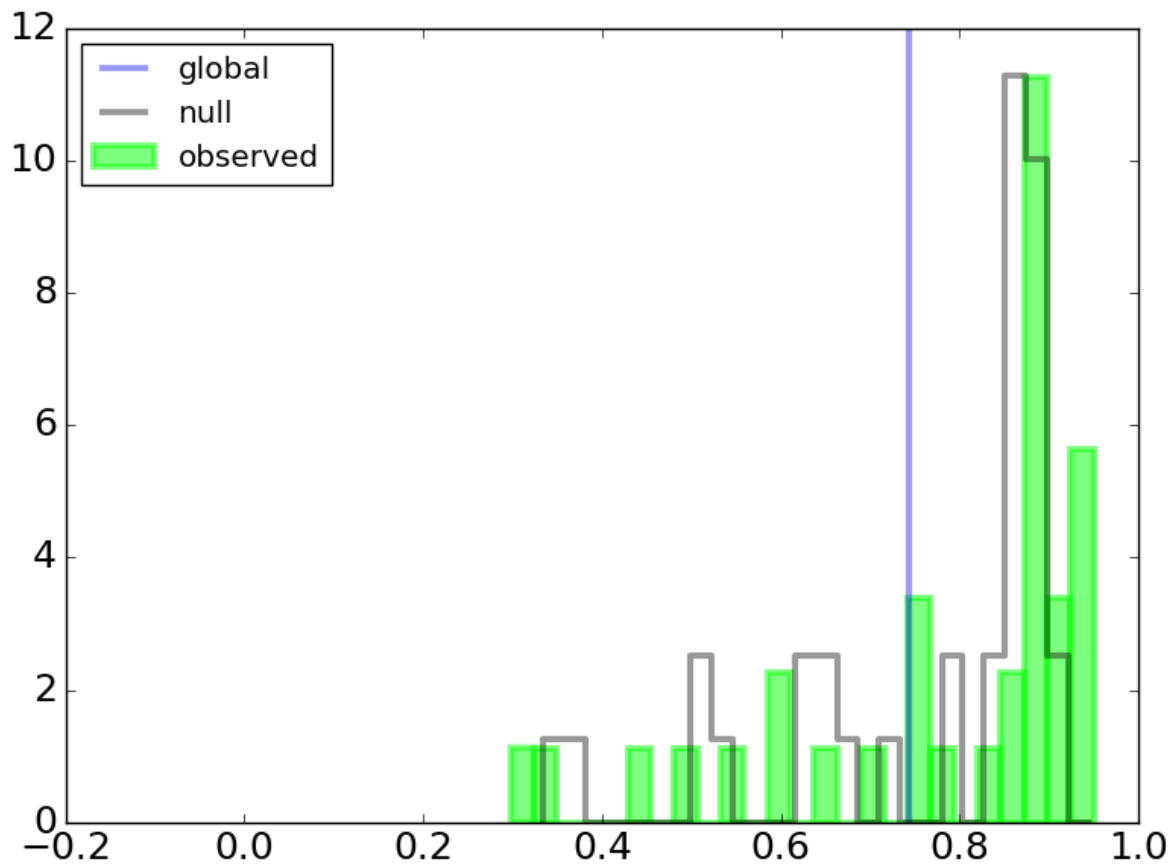
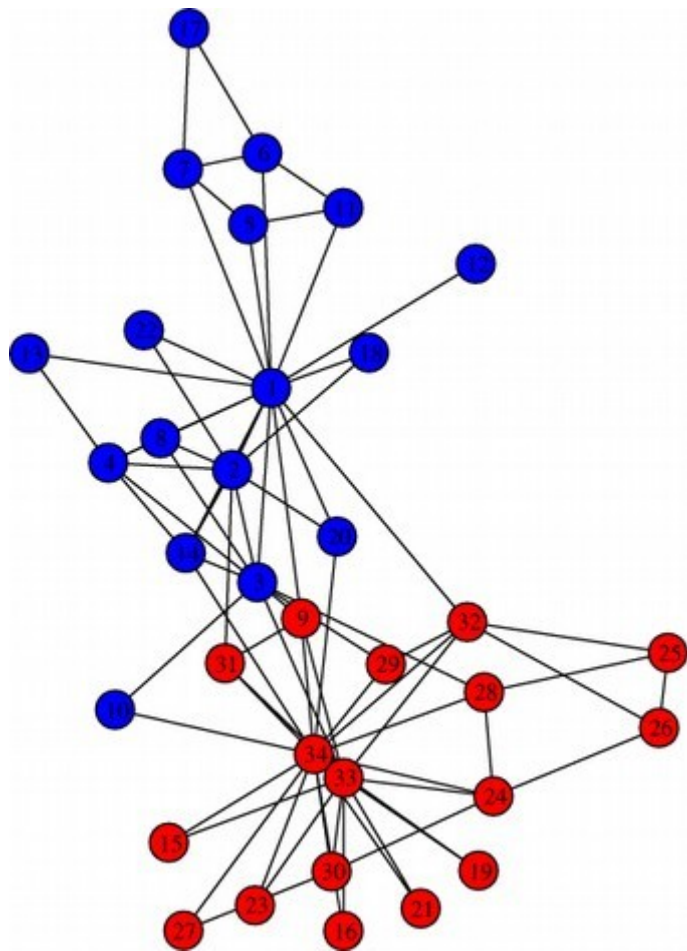






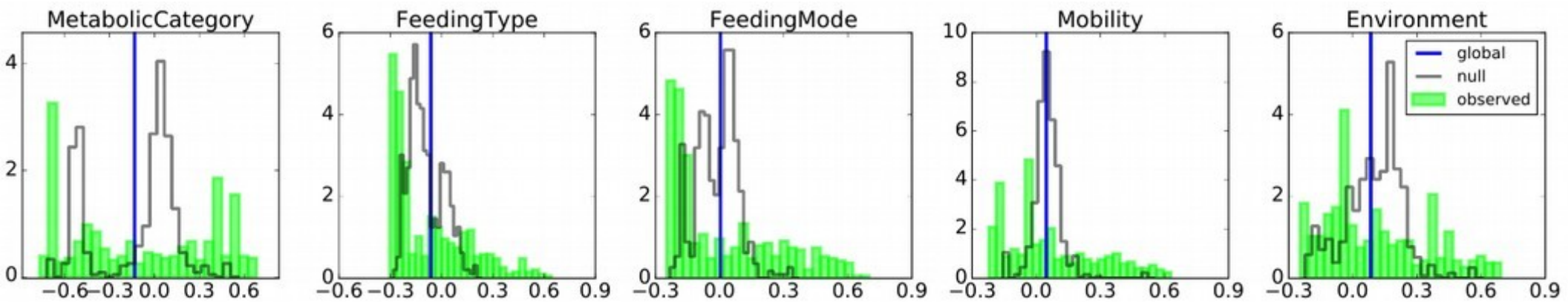


# Karate club - factions

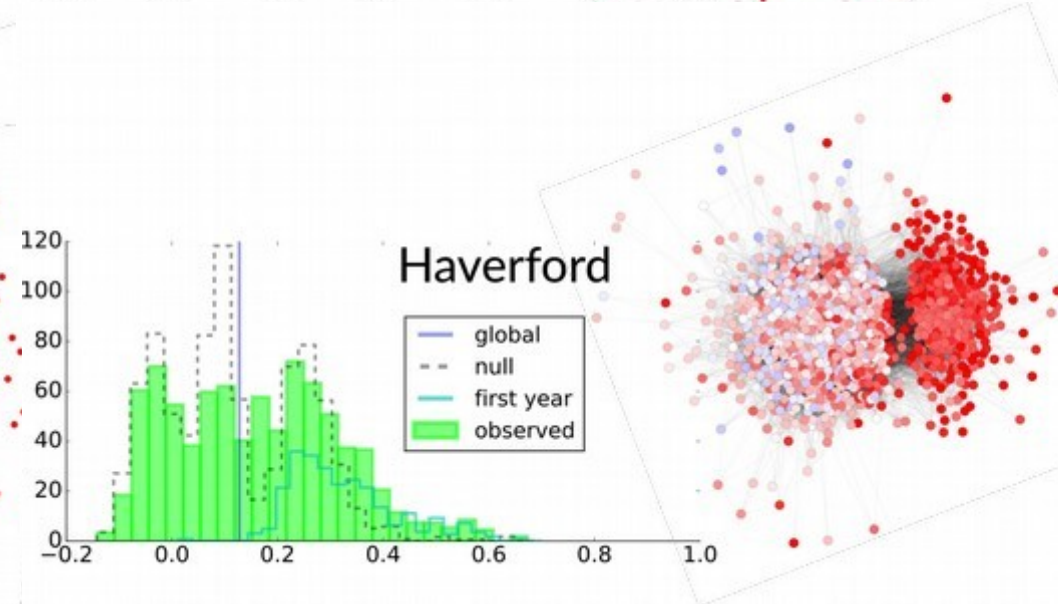
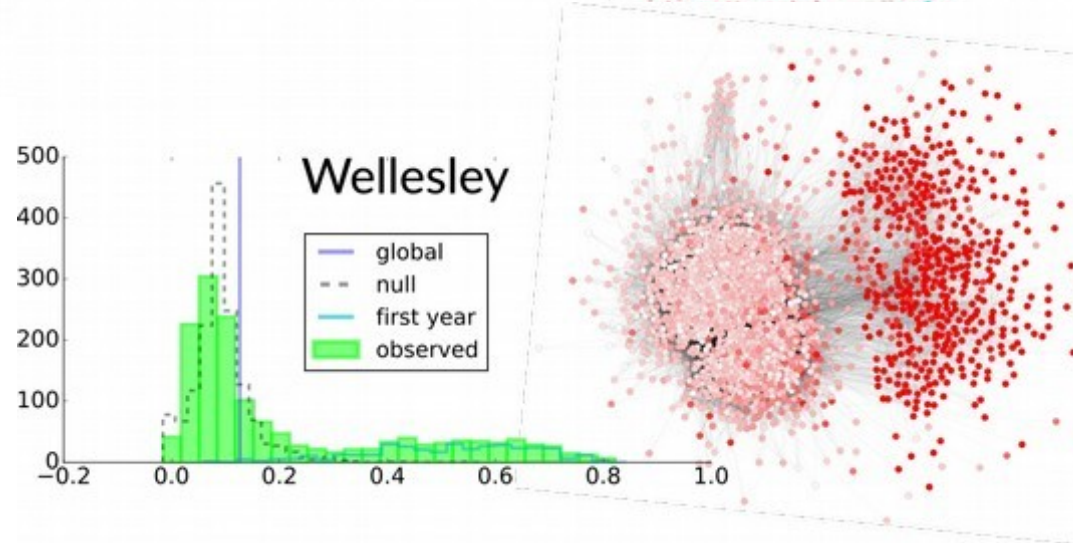
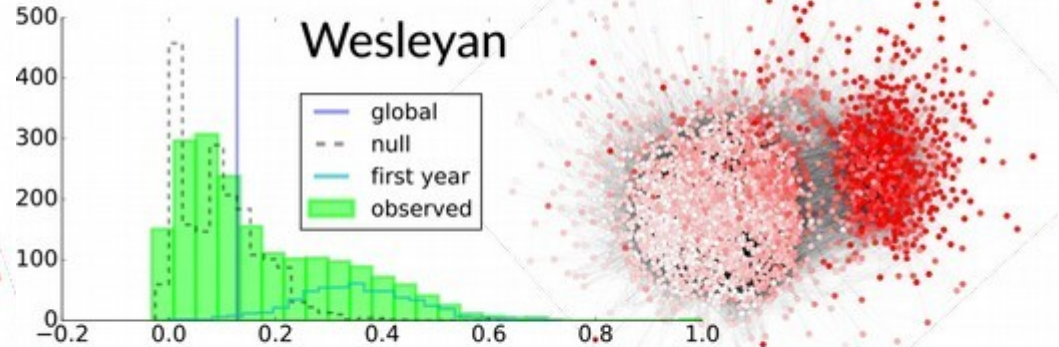
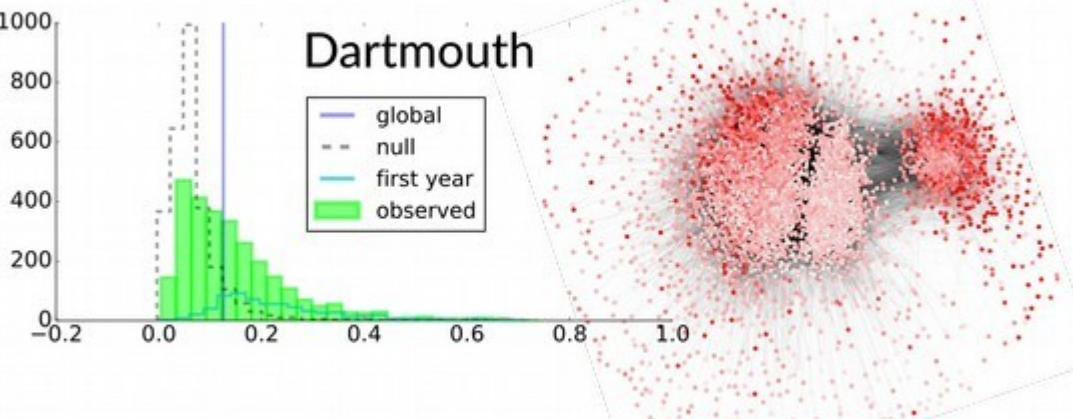




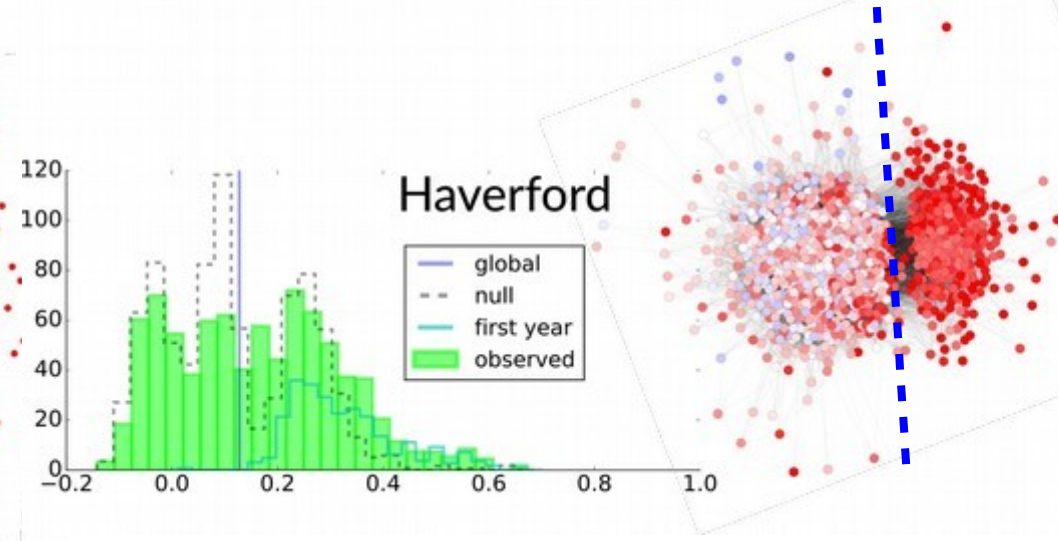
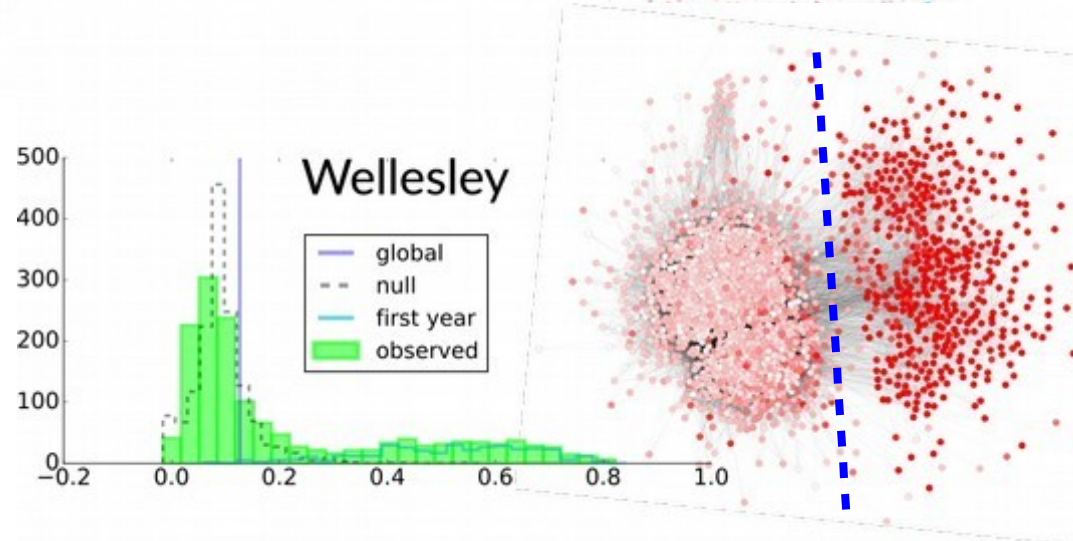
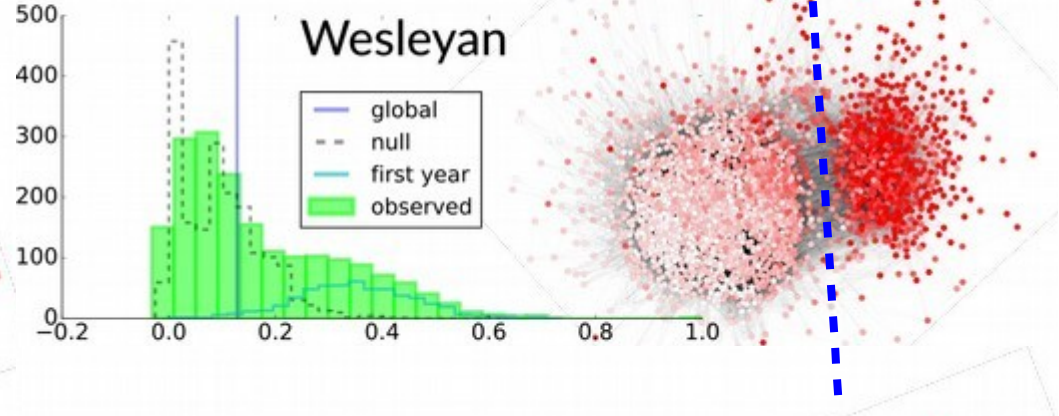
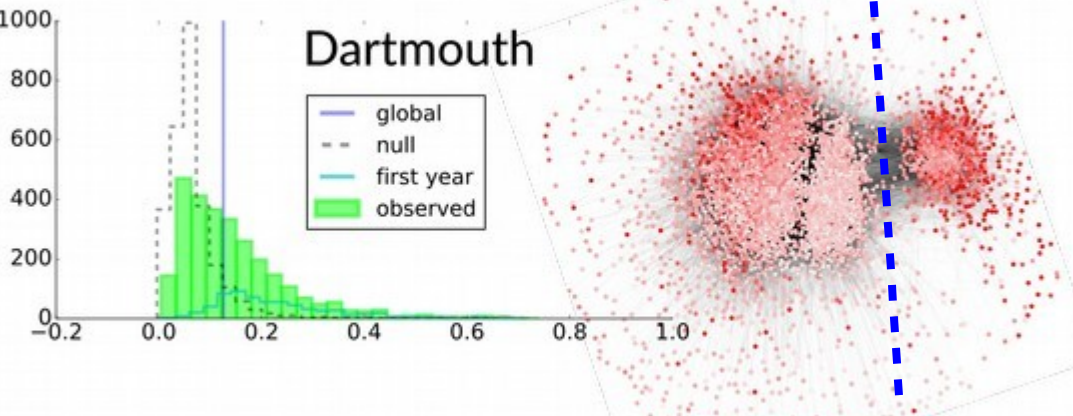
# Weddell Sea food web



# Facebook 100 – residence



# Facebook 100 – residence



# Summary

Assortativity is a correlation coefficient

- can lead to ambiguity in its interpretation

Often we observe variation in the assortativity across the network

- In some cases its not possible to have constant assortativity

Our framework provides a means to compute global network measures within a local neighbourhood



# Questions?



Jean-Charles  
Delvenne



Renaud  
Lambiotte

## Multiscale mixing patterns in networks

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<sup>2</sup>*naXys, Université de Namur, Namur, B-5000, Belgium*

<sup>3</sup>*CORE, Université catholique de Louvain, Louvain-la-Neuve, B-1348, Belgium*

<sup>4</sup>*Mathematical Institute, University of Oxford, Oxford, UK*

pre-print arXiv:1708.01236

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